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Abstract

My dissertation studies the relations between macroeconomic quantities and asset prices. The first chapter takes a production-based approach and investigates how different types of business investment are linked to stock returns. The second chapter takes a consumption-based approach and investigates how the interaction between limited enforcement and preference heterogeneity affects individual consumption, risk sharing and asset prices.

In Chapter One "Capital heterogeneity, time-to-build, and return predictability", I study how two major types of business investment, equipment and structures, are differently linked to stock returns. I empirically show that the investment rate of equipment has a significantly stronger predictive power for stock returns than the investment rate of structures, both in-sample and out-of-sample, using US aggregate-, US asset-, US industry-, and UK aggregate-level data. To explain this empirical finding, I build a quantitative general equilibrium production model in which it takes a shorter time-to-build for equipment investment than for structures investment to transform into productive capital. In the model, equipment investment reacts to productivity shocks in a more timely manner, and thus it reflects more of the information contained in stock prices. In addition, the model provides theoretical support for previous empirical findings of return predictability from planned investment.

In Chapter Two "Asset pricing and risk sharing with limited enforcement and heterogeneous preferences", I introduce heterogeneous preferences (heterogeneity in risk aversion and time discount factor) into a two-agent endowment economy with enforcement constraints and aggregate and idiosyncratic income risk (Alvarez and Jermann (2001)), and study the corresponding asset pricing and risk sharing implications. I show that the relative time discount factor and the interaction between heterogeneous risk aversion and aggregate risk affect the evolution of the relative Pareto weight of agents over time. I demonstrate that preference heterogeneity can generate a positive equity premium with only idiosyncratic risk present, since the conditional pricing kernel is time-varying depending on which agent is the marginal pricer. I use a recursive Lagrangian method to solve a calibrated model and show that preference heterogeneity boosts the mean and volatility of equity premium quantitatively, when the more risk averse and/or the more patient agent cannot trade away most of his income risk with the other agent because of enforcement constraints.

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Chapter 1

Capital Heterogeneity, Time-To-Build, and Return Predictability

1.1 Introduction

Neoclassical theory of investment suggests that when discount rates fall, the marginal benefit of investing increases, and thus real investment should rise.[1] This implies that *aggregate* investment should negatively predict future stock *market* returns (e.g., Cochrane (1991)). This negative prediction may become weak if there are investment lags (e.g., delivery lag, planning lag, or construction lag), since in that case aggregate investment will not change immediately when discount rates change. However, the components of aggregate investment are heterogeneous in investment lags. Investment types with shorter lags could, potentially, still respond quickly to changes in discount rates and preserve the strong negative predictions for future stock market returns.

In this paper, I study how *differently* the two major types of business investment, equipment (e.g., machines) and structures (e.g., factories), are associated with future stock market returns. It is widely believed that structures investment requires a longer time to complete

[1] For surveys in investment, see Jorgenson (1971), Abel (1990b), Chirinko (1993), Caballero (1999), and Bond and Van Reenen (2007).

than equipment investment: It takes about two years to plan and build a manufacturing plant, and only one to two quarters to deliver industrial equipment. This shorter investment lag could tie equipment investment more closely than structures investment to future stock market returns.

Using aggregate investment data from the US Bureau of Economic Analysis (BEA), I find that the investment rate of equipment predicts future stock market returns significantly better than the investment rate of structures, both in-sample and out-of-sample. The 20-quarter prediction R^2, for equipment versus structures, is 39% versus 8% in-sample and 35% versus negative out-of-sample. Also the prediction coefficient is significantly negative for equipment, and negative but insignificant for structures. This stronger predicting power of equipment is further borne out by disaggregated US asset- and industry-level data, and UK aggregate-level data. Disaggregated US investment data reveal an interesting phenomena: Assets with long investment lags, such as structures investment in petroleum and natural gas and in railroad transportation, could even display *positive* prediction coefficients for future stock market returns.

Equipment investment and structures investment also show different patterns of business cycle fluctuations. Equipment investment comoves with total factor productivity (TFP), but structures investment lags TFP for four quarters.[2] This suggests that equipment investment responds to TFP fluctuations more quickly than structures. Fluctuations in TFP are a key underlying economic force for movements in discount rates. In good economic times, TFP is high and aggregate risk is low, and vice versa. Thus, the quicker response of equipment investment to TFP fluctuations could lead to equipment's tighter linkage to future stock market returns.

To verify this, I build a general equilibrium production model with TFP shock as the driving force of the economic fluctuations. I use a time-to-build (TTB) specification (Kydland and Prescott (1982)) from macro literature to capture investment lags. The key model assumption is that structures investment has a longer TTB than equipment investment (5 quarters for structures versus 1 quarter for equipment), with most resources required in later stages or so-called time-to-plan (TTP; see Christiano and Todd (1996)). In the model,

[2]Structures investment also lags GDP more quarters than equipment investment does.

in addition to the difference in TTB, equipment is different from structures in several other respects. Equipment depreciates faster,[3] has a higher factor share in aggregate production,[4] and has a potentially different adjustment cost.[5,6] The model can generate the comovement between equipment investment and TFP and the lagging behavior of structures investment to TFP, as in the data. The model also produces the stronger power of equipment investment than structures investment for predicting future stock returns, consistent with this paper's main empirical finding. I show that only heterogeneous TTB, among all of the heterogeneities, is necessary for these model predictions.

The model works as follows. When a positive TFP shock hits the economy, equipment investment and the stock price increase immediately, and the expected stock return falls. But due to TTB along with TTP, structures investment has small increases initially and big rises in later periods. The delayed response of structures investment results in its lagging behavior to TFP. It also causes its weaker performance for return prediction, because today's structures investment has not fully absorbed the good news already reflected in stock markets.

The model produces satisfactory macro quantities and asset prices. Consumption is less volatile than output, while investment fluctuates much more than output. The equity risk premium is high and volatile (4.28% mean and 15.01% volatility for unlevered returns). To achieve this good fit, I have followed Chen (2017) and introduced external habit preference (Campbell and Cochrane (1999)) and high capital adjustment costs into the model. Since external habit preference gives rise to large fluctuations in discount rates, a *positive* TFP shock in the model acts, essentially, as a *negative* discount rate shock. When a positive TFP

[3]The slower depreciation of structures is positively related to its longer TTB, as stated in Prescott (2016), as follows: "Stocks of capital lagged output, with the lag increasing with the durability of the capital. Inventory stock was almost contemporaneous, producer durables stocks lagged a few quarters, and structures lagged a couple of years".

[4]See Valentinyi and Herrendorf (2008).

[5]Israelsen (2010) uses GMM estimation and finds a higher adjustment cost curvature for equipment than structures. The opposite is assumed in the calibration in Jermann (2010).

[6]There are other differences between equipment and structures that I do not model. Equipment investment has higher tax benefits (House and Shapiro (2008)); the relative price of equipment investment to consumption has been declining, while the relative price of structures investment to consumption has been increasing (Greenwood et al. (1997); Jones (2016)); equipment capital complements skilled labor, which structures capital substitutes for (Krusell et al. (2000)); equipment investment contributes to economic growth more (De Long and Summers (1991)); equipment can be either purchased from abroad or produced domestically, while structures cannot be purchased from abroad (House et al. (2017)).

shock hits the economy, the stock price and stock return rise on impact, but the dividend falls. The future stock price has to fall to accommodate the fall in the dividend. To verify this mechanism more formally, I follow Campbell and Shiller (1988) and decompose the dividend-price ratio into discount rates (long-run stock returns) and cash flows (long-run dividend growth). By using VAR (vector autoregression) analysis, I find that discount rates indeed drive almost all of the variation in the dividend-price ratio in the model. This confirms that the return predictability from investment in the model comes from discount rate variations as in the data.[7]

The model assumption of longer TTB in structures than equipment is consistent with the empirical evidence. First, this assumption produces the right lead-lag relations between investment and TFP, as in the data. The 5-quarter TTB for structures gives rise to the 4-quarter lag of structures investment to TFP, while the 1-quarter TTB for equipment causes equipment investment to comove with TFP. Second, this assumption is consistent with direct evidence from economic surveys. Using the Census Bureau's Survey of Manufacturers' Shipments, Inventories, and Orders, Jones and Tuzel (2013a) show that the delivery lag (approximated by the ratio of unfilled orders to shipments) is about 2-6 months for durable equipment.[8] Based on the Census Bureau's Survey of Construction Spending, also known as the Value of Construction Put in Place Survey, Montgomery (1995) finds that the value-weighted construction length of time for nonresidential structures projects is 16.7 months over the period 1961-1991. I update this statistic and find that the construction length is 13.6 months over the sample 2001-2015.[9]

[7]TFP drives both discount rates and cash flows in the model. Hypothetically, it is possible that cash flows drive the variation in the dividend-price ratio and correlate negatively with discount rates. High investment today that predicts lower future discount rates is simply a manifestation for predicting higher future cash flow growth.

[8]In detail, the delivery lags are 1.99, 2.44, 3.28, 2.93, and 6.22 months, respectively, for primary metal, fabricated metal, industrial machinery, electronic equipment, and transportation equipment.

[9]For further evidence of TTB, Mayer (1960) finds that the average time for nonresidential structures between the decision to undertake the project and the completion of construction is 7 quarters. Jorgenson and Stephenson (1967) find the investment lag to be 6 to 12 quarters for manufacturing industries. Koeva (2000) uses Lexis-Nexis news data and finds Compustat firms' plant construction time to be around 2 years in most industries. Lettau and Ludvigson (2002) find indirect evidence for investment lags from the prediction patterns of risk premium proxies for investment growth across horizons. For further evidence of longer TTB for structures than equipment, Abel and Blanchard (1988) find that it takes on average 1 year to build an industrial structure, while it takes about 6 months to receive equipment. Boca et al. (2008) use a panel of Italian firms to estimate a structural heterogeneous TTB model and find that TTB for equipment is 4 quarters, while TTB for structures is 2 to 3 years.

This paper provides theoretical support for previous empirical findings of return predictability from *planned* investment, as in Lamont (2000) and Jones and Tuzel (2013b).[10] In the model, although the structures investment expenditure does not predict returns, the structures investment decision or planned structures investment does predict. Consistent with Lamont (2000), both the growth of planned structures investment and the planned structures investment rate negatively predict future market returns. I also construct the ratio of planned structures investment to structures investment expenditure analogous to Jones and Tuzel (2013b)'s ratio of nonresidential building starts to structures investment expenditure (Starts/SI). I find that my ratio shows the highest predicting R^2 for annual market returns. However, Starts/SI displays large predicting power at long horizons from 5 to 7 years. This difference could be due to the inclusion of government structures investment in Starts/SI. Belo and Yu (2013) show that government investment is negatively correlated with private investment and positively predicts future market returns. Further decomposition of government investment into equipment and structures shows that equipment predicts returns positively at all horizons, while structures predicts negatively at long horizons. Thus, it is possible that government structures counteracts the negative prediction of private structures at short horizons, but reinforces it at long horizons.[11]

This paper contributes to the asset pricing literature that studies the heterogeneities between equipment and structures. Tuzel (2010) emphasizes the slower depreciation of structures than equipment and shows that firms with more real estate holdings suffer more from bad productivity shocks and are riskier on average. Jermann (2010) and Israelsen (2010) model equipment and structures as two types of capital with different prices, adjustment costs, and depreciation rates, and investigate asset valuations from the producer's first-order conditions. This paper concentrates on another dimension of heterogeneity, i.e., TTB, and studies its implications for asset prices and economic fluctuations. In particular, I find that TTB reduces the elasticity of structures-capital supply and dampens the fluctuation in structures investment. Thus, we do not necessarily need a higher capital adjustment

[10] A recent paper by Li et al. (2017) shows that a bottom-up measure of aggregate investment plans also predicts future stock market returns.

[11] In addition, the strong positive prediction of government equipment for returns could also contaminate Jones and Tuzel (2013b)'s new orders to shipment ratio (NO/S), which shows predictability only at short horizons up to 1 year.

cost for structures, as in Tuzel (2010), to match the lower volatility of structures investment, compared to equipment. In fact, equipment and structures have the same adjustment cost in my benchmark calibration, while their volatilities are well matched to data.

This paper contributes to the literature on the implications of TTB for macro quantities and asset prices. Kydland and Prescott (1982) is the first to show that TTB plays an important role in shaping business cycle fluctuations. Altuğ (1993) shows that when there is TTB, the marginal investment q does not equal the average investment q.[12] A closely related paper is Kuehn (2009). Kuehn brings TTB to asset pricing and demonstrates that TTB can explain the negative correlation between investment growth and stock returns as found in the data.[13] In Kuehn's model, there is a single type of capital with two-period TTB, and utility is constant relative risk aversion (CRRA), which does not generate a large enough risk premium. My model is more complex with two types of capital, more periods of TTB, and external habit preference. It generates realistic asset prices and is suitable for running the return predictability tests I focus on. In addition, as noted in Rouwenhorst (1991), the impulse responses to TFP shocks oscillate for a TTB model with a single type of capital and no adjustment costs. This is inconsistent with the empirical evidence. Kuehn shows that adding *investment* adjustment cost can make the impulse responses become smooth, but adding *capital* adjustment cost does not work. However, in my model, even when there is no adjustment cost, the impulse responses are smooth due to the assumption of two types of capital. Equipment with the standard 1-quarter TTB can absorb TFP shocks upfront. The supply of overall capital (equipment plus structures) is elastic in the short run, although the supply of structures capital is not. The assumption of a single type of capital also leads to a counterfactual negative correlation between consumption growth and investment growth when TTP is strong in Kuehn's model. However, my model still produces a positive correlation as in the data, since equipment investment comoves with consumption.

[12] For other TTB implications in macro literature, see Altuğ (1989), Rouwenhorst (1991), Christiano and Todd (1996), Wen (1998), Zhou (2000), Gomme et al. (2001), Christiano and Vigfusson (2003), Millar (2005), Casares (2006), Edge (2007), Lucca (2007), Kalouptsidi (2014), Bornstein et al. (2017), among others.

[13] In addition, Chen (2016) demonstrates that TTB generates procyclical dividends and increases the risk premium. TTB has also been applied to studies of capital structure and investment-cash flow sensitivity. Tsyplakov (2008) finds that smaller firms have longer TTB and may explain the leverage differences between small and large firms. Tsoukalas (2011) shows that TTB helps to explain investment-cash flow sensitivity.

This paper also contributes to the literature that links investment to stock returns (see Kogan and Papanikolaou (2012) and Zhang (2017) for an overview). Cochrane (1991) shows that the stock return should equal the investment return (see also Restoy and Rockinger (1994)) and finds empirical support in aggregate time-series data. Cochrane (1996) tests aggregate investment growth as a risk factor for the cross section of stock returns. Liu et al. (2009) extend Cochrane (1991) to test the equivalence between the stock return and the investment return at the level of individual firms, and find some supporting evidence. The literature of cross sectional asset pricing has shown that firms with high investment today have lower subsequent average stock returns (see portfolio sorts on growth in investment-sales ratio in Titman et al. (2004), on investment growth in Anderson and Garcia-Feijóo (2006), on investment rate in Xing (2007), on asset growth in Cooper et al. (2008), on inventory growth in Belo and Lin (2011), and on investment rate in brand capital in Belo et al. (2014b)).[14] Hou et al. (2015) and Fama and French (2016) include an investment factor in their four-factor and five-factor asset pricing models, respectively, to explain the wide range of cross-sectional asset pricing anomalies. In addition, a strand of literature on production-based asset pricing models—in either general-equilibrium approach or partial-equilibrium approach with an exogenously specified stochastic discount factor—studies how firms' investment decisions affect the cross-section of stock returns. An incomplete list of contributions include Berk et al. (1999), Kogan (2001), Gomes et al. (2003), Carlson et al. (2004), Kogan (2004), Zhang (2005), Cooper (2006), Ai and Kiku (2013), Kogan and Papanikolaou (2013), and Kogan and Papanikolaou (2014). Also, several papers, namely Cochrane (1988), Cochrane (1993), Belo (2010), and Jermann (2010), develop alternative production technologies to recover the stochastic discount factor from the marginal rates of transformation inferred from producers' first-order conditions, to directly link investment to stock returns without consumption.

This paper is related to the asset pricing literature studying general equilibrium production models. This literature demonstrates that it is difficult for standard production models to match both business cycle and asset pricing moments (see Jermann (1998) and

[14] Relatedly, firms' hiring is like investment when there are labor adjustment costs. Belo et al. (2014a) show that firms with higher hiring rates also have lower average future stock returns.

Boldrin et al. (2001), who use *internal* habit preferences (e.g., Abel (1990a); Constantinides (1990))).[15] Chen (2017) improves over the previous models by introducing *external* habit preference (Campbell and Cochrane (1999)) to the standard production model and shows that a low intertemporal elasticity of substitution paired with large capital adjustment cost can generate a high equity premium and a high investment volatility while giving a low volatility of the risk-free rate. Chen (2017) also shows that the investment rate can predict stock returns in his model as in the data. I introduce two types of capital—equipment and structures with heterogeneous TTB—into his single-capital model. I find that TTB dampens the volatility of structures investment, delays the responses of structures investment to productivity shocks, and weakens the predicting power of structures investment for stock returns. My TTB model shares some similarities with the two-sector model with factor immobilities in Boldrin et al. (2001). In both models, capital supply is inelastic in the short run.[16] This leads to consumption overshooting and the "inverted leading-indicator property of interest rates" as in the data. Consumption volatility is usually too high in this type of models featuring inelastic short-run capital supply. But because there are two types of capital in my model, equipment investment can absorb the productivity shocks on impact in addition to consumption. Thus my model can deliver a realistic consumption volatility.

This paper is also related to the vast literature on time-series return predictability (see Lettau and Ludvigson (2010) and Koijen and Van Nieuwerburgh (2011) for an overview), and in particular the predictability of macro quantities (such as output, consumption, investment, and labor) for stock returns. Cochrane (1991) and Lamont (2000) show that investment predicts stock returns. I show that equipment investment is more tightly linked to future stock returns than structures investment. Other macro predictors include the

[15]See Kogan and Papanikolaou (2012) for an overview. See the seminal Mehra and Prescott (1985); the early Tallarini (2000); papers related to long-run consumption risk à la Bansal and Yaron (2004): Kaltenbrunner and Lochstoer (2010), Campanale et al. (2010), Ai et al. (2013), Croce (2014), Kung and Schmid (2015), Ai et al. (2017); papers related to rare disasters à la Barro (2006): Gourio (2012); papers related to investment shocks: Papanikolaou (2011), Garlappi and Song (2017); papers related to labor frictions: Danthine and Donaldson (2002), Favilukis and Lin (2015), and Petrosky-Nadeau et al. (2017); and papers related to technology innovation and competition: Gârleanu et al. (2012a), Gârleanu et al. (2012b), Bena et al. (2016), Gârleanu et al. (2016), Corhay et al. (2017), Gofman et al. (2017), and Kogan et al. (2017); among others.

[16]In my model, the supply of structures capital is inelastic in the short run due to TTB, but the supply of equipment capital is partially elastic under adjustment costs.

consumption-wealth ratio (CAY; Lettau and Ludvigson (2001)), the consumption-labor income ratio (Santos and Veronesi (2006)), the output gap (Cooper and Priestley (2009)), the employment growth (Chen and Zhang (2011); Belo et al. (2017)), the ratio of new orders to shipments of durable goods (Jones and Tuzel (2013b)), the expected investment growth (Li et al. (2017)), and the government debt-output ratio (Liu (2017)), etc.

This paper is structured as follows. Section 2 describes the data, defines the variables used, presents summary statistics, and shows the empirical specifications and results. Section 3 sets up the model and derives theoretical implications. Section 4 presents calibration and quantitative predictions, and Section 5 concludes.

1.2 Empirical Evidence

In this section, I first describe the data dealings and constructions for the main variables—the investment rates of equipment and structures—at aggregate level, asset level, industry level, and international level. I then present the summary statistics. Next, I provide evidence of longer TTB for structures than for equipment. Last, I specify the predictive regressions of investment rates for risk premia and present the empirical results and note in particular that the investment rates of equipment predict risk premia better than the investment rates of structures.

1.2.1 Data

I follow Cochrane (1991) and construct the time series of the investment-capital ratio or investment rate (IK) using the following recursion derived from the perpetual inventory method:

$$IK_t = \frac{I_t}{I_{t-1}} \frac{IK_{t-1}}{1 - \delta + IK_{t-1}}. \tag{1.1}$$

The initial value of the investment rate is set to be the steady-state level, i.e., the depreciation rate plus the average investment growth rate, $IK_0 = \delta + E(I_t/I_{t-1})$. Given the initial value, the whole time series of the investment rate can be derived from the above recursion.

I use quarterly investment data from BEA National Income Product Accounts (NIPA) tables and annual depreciation rates implied from BEA Fixed Assets (FA) tables. I use one-fourth of annual depreciation rates as quarterly rates. The sample period is from 1947 quarter 1 to 2015 quarter 4. Quarterly private nonresidential real equipment and structures investment is from nominal values in *NIPA Table 1.1.5* line 11 (equipment) and line 10 (structures) deflated by corresponding price indexes in *NIPA Table 1.1.4*. In NIPA, total private nonresidential investment includes equipment, structures, and intellectual property and products (IPP). Since this paper focuses on equipment and structures, I exclude IPP for convenience and consistency.[17] To construct a series for real total nonresidential investment without IPP, I apply the Fisher formula to equipment and structures.[18]

I calculate annual depreciation rates as the time-series averages of the ratio of real depreciation to last yearend real capital stock. The real capital stock series for equipment and structures are the nominal capital stocks of base year 2009 in *FA Table 1.1* line 5 (equipment) and line 6 (structures) multiplied by the corresponding chain-type quantity indices in *FA Table 1.2* and scaled by 100. The real depreciation series for equipment and structures are constructed similarly with nominal stocks in *FA Table 1.3* and chain-type quantity indexes in *FA Table 1.4*. I apply the Fisher formula again to obtain the real capital stock and real depreciation of total nonresidential capital without IPP. Annual estimates for depreciation rates of nonresidential total, equipment, and structures are, respectively, 5.04%, 10.90%, and 3.17%.[19]

I construct quarterly disaggregated nonresidential equipment and structures investment rates at asset level. BEA disaggregates nonresidential equipment into information processing equipment, industrial equipment, transportation equipment, and other equipment, and nonresidential structures into commercial and health care; manufacturing; power and communication; mining exploration, shafts and wells; and other structures. I apply the same

[17]Including IPP has little effect on empirical results; see Appendix A.5.

[18]The Fisher formula for the growth rate of nonresidential total from time $t-1$ to t is $\sqrt{\frac{\sum p_{t-1}q_t}{\sum p_{t-1}q_{t-1}} \times \frac{\sum p_t q_t}{\sum p_t q_{t-1}}}$, where p's and q's represent price indices and real quantities of equipment and structures. See Bureau of Economic Analysis (2016) for how BEA constructs aggregate estimates from detailed components.

[19]Note that directly using current-cost measures will generate a higher depreciation rate for equipment and a slightly lower depreciation rate for structures as the relative price of equipment has been declining over the sample and the relative price of structures has increased a little. Current cost measures capture both physical wear and economic obsolescence, while real cost measures account for only physical wear. See Jermann (2010), who estimates depreciation rates in current cost measures over the sample 1947-2002 for equipment and structures to be 13.06% and 2.7%, respectively. After adjusting prices, he obtains 11.2% and 3.1%.

perpetual inventory method in equation (1.1). I use investment data from *NIPA Table 5.3.4 and 5.3.5*, and calculate implied depreciation rates from *FA Table 2.1, 2.2, 2.4, and 2.5*. The data sample is from 1947Q1 to 2015Q4 for equipment assets and from 1959Q1 to 2015Q4 for structures assets, due to the absence of data for early years.

I also construct annual disaggregated equipment and structures investment rates at industry level.[20] I use BEA 19 industries classified by the three-digit 2012 North American Industry Classification System (NAICS). I apply the same perpetual inventory method as in equation (1.1). I use investment data from *FA Table 3.7E, 3.7S, 3.8E, and 3.8S*, and calculate implied depreciation rates from *FA Table 3.1E, 3.1S, 3.2E, 3.2S, 3.4E, 3.4S, 3.5E, and 3.5S*. At the industry level, BEA reports only total investment of nonresidential and residential, and does not report them separately. This data limitation introduces the effect of residential investment to the industry-level analysis. However, residential investment is mostly reflected in the real estate sector and has little effect on other sectors. To mitigate the effect of residential investment, I drop the real estate industry. I also drop finance and utilities, following the standard practice in the literature. In addition, I drop two industries—management of companies and enterprises and educational services—due to limited data on stock returns. This leaves 14 industries for analysis.

The data for total factor productivity (TFP) is from John Fernald's website, "dtfp". Real gross domestic product (GDP) is the nominal value in *NIPA Table 1.1.5* line 1 deflated by the corresponding price index in *NIPA Table 1.1.4*. The data for nominal aggregate stock market returns and the risk-free rate is from Kenneth French's website. Real returns are nominal returns deflated by seasonally adjusted consumer price index for all urban consumers from the Bureau of Labor Statistics.

Industry-level returns are calculated from the Center for Research in Security Prices (CRSP) and Compustat. I use monthly stock returns from CRSP, and correct the delisting bias following the approach in Shumway (1997). I include firms with common shares (shrcd=10 and 11) and firms traded on the NYSE, AMEX, and NASDAQ (exchcd=1, 2, and 3). I use Standard Industrial Classification (SIC) and NAICS from the CRSP/Compustat Merged Annual Industrial Files. Firms are assigned to BEA industries based on their

[20] Industry-level data are not available at quarterly frequency.

NAICS. If a firm's NAICS is not available, it is set to be the most frequent 3 digit NAICS based on the firm's SIC.[21] The industry risk premium is calculated as the difference between the value-weighted returns for all firms in that industry and the risk-free rate. The sample is annual from 1962 to 2015.

I construct UK aggregate investment rate series for nonresidential equipment and structures, using the perpetual inventory method in equation (1.1). Quarterly investment data from 1970Q1-2013Q4 are downloaded from *"gross fixed capital formation by 6 asset types" (namq_pi6_k)* in the Eurostat database. Nonresidential equipment is the aggregate sum of N11131 transport equipment and N11132 other machinery and equipment, while nonresidential structures is N1112 other buildings and structures. Data for returns are from Kenneth French's and John Campbell's websites and International Monetary Fund (IMF) International Financial Statistics. See Appendix A.3 for more details.

1.2.2 Descriptive Statistics

Table 1.1 reports the descriptive statistics of investment rates at aggregate level, asset level and industry level. Panel A shows statistics for quarterly aggregate investment rates. Equipment shows a higher depreciation rate than structures, 2.72% vs 0.79%. Equipment IK has a higher mean (3.88%) and volatility (0.49%) than structures IK (1.35%, 0.25%), while structures IK is slightly more persistent than equipment IK, 0.99 vs. 0.97. Equipment IK highly correlates with total nonresidential IK (0.93), but has a relatively small correlation with structures IK (0.26).

[Insert Table 1.1 about here]

[Insert Figure 1.1 about here]

Figure 1.1 depicts the time series of quarterly aggregate investment rates, which are procyclial. However, structures IK is less procyclical than equipment IK, such as in the recessions of the mid-1950s and early 1960s; structures IK actually increases over the two recessions. Structures IK also shows delayed responses in the 1981-1982 recession and the

[21] In rare cases, there is no NAICS match for the firm's SIC, and the SIC-NAICS concordance tables from the US Census Bureau are used.

recent Great Recession of 2007-2009. There are small increases for structures IK at the beginning of the two recessions before it falls; in contrast, equipment IK falls immediately once the recession starts. The correlation between Hodrick-Prescott (HP; Hodrick and Prescott (1997)) filtered IK and HP-filtered log GDP is 0.81 for equipment and 0.48 for structures (not tabulated).

Table 1.1 Panel B shows the statistics for quarterly asset-level investment rates of equipment and structures. *Information processing equipment* has the highest mean (5.69%) , volatility (0.95%), and correlation with aggregate nonresidential (0.87) among all of the asset types. This conforms to the rise of information and communications technology in the economy over the post-war sample.[22] *Mining exploration, shafts, and wells* shows the lowest correlations among all asset types: 0.21, 0.03, and 0.31, with aggregate nonresidential, equipment, and structures respectively. This is likely because among all of the structures types, mining structures capital depreciates the fastest (1.91%) and the net investment rate (gross net of depreciation) of mining is the smallest (0.24%).

Table 1.1 Panel C shows the statistics for annual industry-level investment rates of equipment and structures. First, industry equipment displays faster depreciation than industry structures. The lowest depreciation rate among industry equipment—8.93% of *transportation and warehousing* equipment—is still higher than the highest depreciation rate among industry structures, i.e., 7.01% of *mining* structures. Second, industry equipment IKs are all positively correlated with aggregate nonresidential IK and aggregate equipment IK. However, the structures IKs of *health care and social assistance* and *other services except government* are mildly negatively correlated with aggregate nonresidential IK. Puzzlingly, the structure IK of *transportation and warehousing* has a significant negative correlation of -0.41 with aggregate structure IK. The likely reason is that this industry has the second-lowest average gross IK (2.98%) and net IK (0.75%). The industry with the lowest structure IK is *agriculture*, which has 2.17% gross IK and -0.32% net IK. *Agriculture* is the only industry whose structures investment falls behind the depreciation.

[22] See Ward (2017) for the evolution and growth implications of IT sector.

Business-Cycle Properties of Investment

I document that equipment investment is different from structures investment in its business-cycle properties. Equipment investment tends to comove with TFP and GDP, while structures investment tends to lag TFP and GDP for several quarters. Table 1.2 reports the quarterly cross-correlations between nonresidential investment (equipment and structures) and TFP, and between nonresidential investment and GDP. I calculate three types of cross-correlations using first-differenced data, HP-filtered data ($\lambda = 1600$), and bandpass-filtered data (Baxter and King (1999), fluctuations from 6 to 32 quarters), as shown in Panels A, B, and C, respectively.

[Insert Table 1.2 about here]

[Insert Figure 1.2 about here]

The first robust result is that equipment has a significant higher contemporaneous correlation ($i = 0$) with TFP and GDP (ranging from 0.42 to 0.80) than structures (ranging from 0.05 to 0.44). In particular, the contemporaneous correlation between structures and TFP is fairly small: 0.13, 0.05, and 0.08 across the three measures. Second, structures lags TFP and GDP more quarters than equipment. Structures lags TFP 3-4 quarters, and lags GDP about 2 quarters, while equipment lags TFP 0-2 quarters, and lags GDP 0-1 quarter. Figure 1.2 shows the correlations between investment growth and TFP growth (i.e., first-differenced data). Equipment investment comoves with TFP, but structures investment lags TFP 4 quarters with increasing correlations from a 1-quarter lag to a 4-quarter lag. The bivariate VAR analysis with TFP growth (ordered first) and investment growth also highlights the lagging behavior of structures investment, as shown in Appendix Figure A.1. Christiano and Todd (1996) show that TTB and TTP help explain the fact that nonresidential investment lags output over the business cycle. Leaning on their findings, I show later in the paper that assuming a longer TTB (along with TTP) for structures than for equipment can generate a longer investment lag for structures than for equipment.

In addition, the positive correlations between structures and GDP stretch into long horizons at 5- and 6-quarter investment lags, where equipment has little or negative correlation to GDP. Take the HP-filtered measure, for example: The correlations between structures and GDP with 5- and 6-quarter investment lags are 0.35 and 0.22, respectively, while the analogs for equipment are 0.03 and -0.13. See also Stock and Watson (1999), who show the cyclicality of various macro quantities and prices, including nonresidential equipment and structures.

1.2.3 Direct Evidence of TTB

Before getting into the main empirical analysis, I provide some direct empirical evidence for a longer TTB for structures than for equipment, which this paper emphasizes.

The source data BEA use to construct series of nonresidential equipment investment is based on the Census Bureau's Survey of Manufacturers' Shipments, Inventories, and Orders. Abel and Blanchard (1988) estimate delivery lags using this survey, along with other datasets, and find that the delivery lags are 2, 2, 3, and 0 quarters for fabricated metals, non-electrical machinery, electrical machinery, and motor vehicles, respectively. Jones and Tuzel (2013a) also use this survey and show that the delivery lag (approximated by the ratio of unfilled orders to shipments) for durable goods is about 4 months. In detail, the delivery lags are 1.99, 2.44, 3.28, 2.93, and 6.22 for primary metal, fabricated metal, industrial machinery, electronic equipment, and transportation equipment.

The source data BEA use to construct series of nonresidential structures investment is based on the Census Bureau's Survey of Construction Spending, also known as the Value of Construction Put in Place Survey. Montgomery (1995) uses the confidential project-level data from this survey of over 52,000 private nonresidential construction projects and finds that the value-weighted construction length of time (LoT) averages 5 to 6 quarters (16.7 months) over the period 1961-1991.[23] Although I do not have access to project-level data, I update the LoT statistic for the recent sample 2001-2015, using publicly available data from the Census Bureau website: see Appendix A.1 for details.

Table 1.3 reports the LoT, or the average number of months from start to completion, for private nonresidential construction projects in 1990-91 and 2001-2015 by value and type of construction. Montgomery (1995) shows that the value-weighted LoT is 15.7 months in

[23]Construction LoT in the language of the Census Bureau, is the same as TTB period in this paper.

1990-91. I find that the value-weighted LoT is 13.6 months over 2001-2015.[24] Although there is a 2- to 3-month decrease over the years, the LoT of 4-5 quarters for nonresidential structures is significantly longer than the delivery lag for nonresidential equipment. The difference is sizable, in terms of the standard quarterly frequency used in the presentation of macro data by statistical agencies and in the calibration of macro models.

[Insert Table 1.3 about here]

The LoT increases with project value, as shown in Panel A. During 2001-15, it takes 20.1 months to complete a project valued at $10,000 thousands or more and 3.9 months for $75-$249 thousands. The equal-weighted LoT across all projects decreases from 14 months in 1990-91 to 7.6 months in 2001-15. This is likely because there are many more small projects in the recent sample, since LoTs across value categories do not change much. This also leads to significantly shorter LoTs across different types in 2001-15 relative to 1990-91, as shown in Panel B. Consistently across different samples and equal- or value- weighted measures, commercial buildings have the shortest LoT. Nevertheless, Millar et al. (2016) find that TTP lags are long for commercial construction projects—about 16 months for the equal-weighted measure and about 26 months for the value-weighted measure.

1.2.4 Empirical Specifications

I use the standard short- and long-horizon predictive regressions (Fama and French (1989)) of the form

$$\sum_{h=1}^{H} R_{t+h} = a + b\ \mathrm{IK}_t + \varepsilon_{t+H}. \tag{1.2}$$

H is the forecast horizon in quarters. $\sum_{h=1}^{H} R_{t+h}$ is the H-period cumulated log excess return for the aggregate stock market or for one industry. R_t is the difference between log aggregate or industry stock return and log risk-free rate. IK_t is the investment rate at

[24] We would expect that there is a significantly shorter LoT for recent sample years, as the technology has improved. Instead, the construction industry has become less productive. One reason is that the industry has become less capital-intensive, with machinery replaced by workers; see Economist (2017): " 'While we are all using iPhones, construction is still in the Walkman [Sony cassette player] phase,' says Ben van Berkel, a Dutch architect. Many building professionals use hand-drawn plans riddled with errors. A builder of concrete-framed towers from the 1960s would find little has changed on building sites today, except for better safety standards."

aggregate, asset or industry level. Both in-sample and out-of-sample tests are performed. For in-sample tests, I report R^2, the regression slope coefficient b, and Newey and West (1987) p values with the correcting lag for standard errors being the number of overlapping periods, $H-1$. For out-of-sample tests, I use the first half of the sample as the training sample, then recursively test and retrain in subsequent periods. I report out-of-sample R^2 relative to historical mean forecasts and the ENC-NEW encompassing test statistic from Clark and McCracken (2001).

1.2.5 Empirical Results

This subsection establishes the empirical finding that the equipment investment rate predicts stock returns better than the structures investment rate with the use of US aggregate-, US asset-, US industry-, and UK aggregate-level data.

How Do Aggregate IKs Predict Aggregate Returns?

Aggregate equipment IK predicts market excess returns better than aggregate structures IK. Table 1.4 reports return predictability results for US investment rates of private nonresidential total, equipment, and structures. Consistent with neoclassical investment theory, all prediction slope coefficients are negative. When discount rates fall, investment should increase. Consistent with Cochrane (1991) and Lamont (2000), nonresidential IK predicts the aggregate risk premium very well, both in-sample and out-of-sample. The R^2 increases over horizons from 1 quarter to 20 quarters, with in-sample R^2 increasing from 3.90% to 39.26% and out-of-sample R^2 increasing from 0.68% to 33.98% at 16 quarters and decreasing to 26.99% at 20 quarters. Equipment IK predicts risk premium as well as nonresidential IK, but structures IK has small in-sample R^2 and negative out-of-sample R^2. This suggests that equipment is the driving component that links nonresidential investment to stock returns.[25] Goyal and Welch (2008) show that the out-of-sample R^2 is usually negative for well-known return predictors, including the dividend-price ratio and the book-market ratio. The strong positive out-of-sample R^2 for IK suggests that IK truly contains useful

[25]Table A.1 shows that the component intellectual property and product of nonresidential investment shows little return predictability. Certainly, IPP has become an important part of nonresidential investment in the recent years. And there is mismeasurement for IPP in BEA data.

information for predicting movements in the risk premium.

[Insert Table 1.4 about here]

[Insert Figure 1.3 about here]

Figure 1.3 shows the actual and predicted future 5-year-ahead risk premium from 1947Q2 to 2011Q1 when the predictor is equipment IK. The predicted in-sample risk premium is countercyclical and captures a significant portion of the variation in the actual risk premium. The predicted out-of-sample risk premium in the second half of the sample almost coincides with the predicted in-sample risk premium. This indicates that the predicting coefficients are fairly stable.

[Insert Table 1.5 about here]

The IK series following Cochrane (1991) are constructed under the assumption of constant depreciation rates.[26] In reality, however, depreciation rates are time-varying. To check the robustness of the results to this assumption, I construct alternative IK series following the method in Bachmann et al. (2013), who use time-varying depreciation estimates from BEA;[27] see Appendix A.2 for details. Table 1.5 reports the return predictability results from these alternative IK series, which are similar to those in Table 1.4. The predicting power for equipment IK is even stronger than nonresidential IK at longer horizons; for example, equipment IK has a 33.71% in-sample R^2 and a 43.21% out-of-sample R^2, while the analogs for nonresidential IK are 27.74% and 19.86%, respectively, at the 20-quarter horizon.

How Do Asset-Level IKs Predict Aggregate Returns?

Does a specific type of equipment or structures drive the predicting difference between aggregate equipment and aggregate structures? Do different types of equipment or structures show significant differences in predicting aggregate risk premium? Table 1.6 answers these questions; it reports predictability results by equipment- and structures-asset types. All

[26] Typical macro models assume this as well.

[27] BEA calculates aggregate equipment and structures investment from detailed asset-level investment data. BEA assumes constant depreciation rates for detailed assets, but due to compositional changes over time, aggregate equipment and structures have time-varying depreciation rates.

of the four types of equipment, i.e., *information processing*, *industrial*, *transportation*, and *other*, predict aggregate risk premium well. Also, structures types generally exhibit lower predicting R^2 than equipment. Therefore, equipment's superior performance to structures in return predictions is not driven by a specific type of equipment or structures asset.

[Insert Table 1.6 about here]

Notably, the investment in *mining exploration, shafts, and wells* has no predicting power and the predicting slope is even *positive* though not significant. This positive slope is driven by the sub-asset type *petroleum and natural gas*.[28] As shown in Bornstein et al. (2017), the average lag between investment and production in the oil industry is 12 years. The long TTB lag makes the investment in oil wells reflect mostly past economic climates and reacts little to future business conditions. Investment in *petroleum and natural gas* is acyclical, and has a contemporaneous correlation of 0.04 with GDP and -0.05 with TFP in growth rates.

How Do Industry IKs Predict Aggregate and Industry Returns?

Industry equipment IK predicts *aggregate* risk premium better than industry structures IK does. Table 1.7 Panel A shows how 14 US industry equipment IK series and structures IK series predict aggregate risk premium at a 5-year horizon. The last column shows that equipment has a higher predicting R^2 than structures for all industries except *mining*. The difference in R^2 can be as large as about 20% for *wholesale* and *transportation and warehousing*. The positive slope of *mining* (2.81) structures IK is reminiscent of the result for structures type *mining exploration, shafts, and wells* in Table 1.6. Consistently, detailed industry-level data show that the *oil and gas extraction* industry drives the positive prediction. The detailed industry-level data also show that the *railroad transportation* industry drives the positive predicting slope (3.45) of *transportation and warehousing* structures IK. This rejoins the idea that investment in assets with long TTB periods may even predict aggregate risk premium positively, though not significantly.

[28] *Mining exploration, shafts, and wells* include two sub-asset types, i.e., *petroleum and natural gas* and *mining*. Mining actually has a negative predicting slope.

In addition, service industries have lower R^2 than traditional industries such as manufacturing; a possible explanation is that service industries are labor-intensive instead of capital-intensive. Fluctuations in labor hiring in these industries, therefore, may be more informative about aggregate economic conditions.

[Insert Table 1.7 about here]

For most industries, equipment IK also captures more *industry* risk premium than structures IK does. Table 1.7 Panel B shows how US 14 industry IK series predict each industry's risk premium. As shown in the last column, equipment IK outperforms structures IK in the sectors *wholesale trade*, *transportation and warehousing*, *information*, and *professional, scientific, and technical services*; the difference of R^2 can be as large as about 26%. Structures IK outperforms equipment IK in the *retail* sector with about 10% R^2 difference.

International Evidence

UK aggregate-level data also show that equipment IK predicts aggregate risk premium better than structures IK does. Table 1.8 reports return predictability results for UK quarterly IK series of nonresidential equipment and structures. At short horizons from 1 quarter to 8 quarters, both equipment and structures have little predictability. As the horizon increases to 16 quarters to 24 quarters, equipment shows significantly higher in-sample and out-of-sample R^2 than structures. For equipment, the in-sample R^2 ranges from 11.12% to 23.46%, and the out-of-sample R^2 is large, from 21.33% to 33.18%.

[Insert Table 1.8 about here]

1.2.6 Linking Time Series and Cross Section

Since nonresidential investment rates predict aggregate risk premium, a natural question is, how do these investment rates predict returns of the established factors that capture the cross-section of stock returns, especially investment factors? Also, how do nonresidential investment rates predict risk premium of the portfolios sorted on the characteristics related to firm risk? Table 1.9 reports prediction results for various factor returns and portfolio risk premium at a 20-quarter horizon, including returns of four factors from Fama and French (2016) 5-factor model (size (SMB), value (HML), investment (CMA), and profitability (RMW)) and risk premia of the extreme decile portfolios (deciles 1 and 10) associated with the factor characteristics, returns of two factors from Hou et al. (2015) four-factor model (investment (IA) and profitability (ROE)), and returns of investment-minus-consumption factor (IMC) from Papanikolaou (2011).[29]

[Insert Table 1.9 about here]

Equipment IK predicts CMA well with 25% in-sample R^2 and 23% out-of-sample R^2, while structures IK has strong predicting power for IA with 46.5% in-sample R^2 and 35% out-of-sample R^2. Therefore, *nonresidential investment is linked to the investment factors that capture cross-sectional stock returns.* Also, the linkages between equipment and CMA and between structures and IA may provide directions for distinguishing the economic forces behind the two investment factors.[30] In addition, the non-predictability for RMW versus moderate predictability for ROE also shows the difference between the two profitability factors.[31,32]

Equipment IK and structures IK predict IMC negatively. When the expected returns for investment firms relative to consumption firms fall, aggregate investment should rise.[33] It is also worthwhile to note that structures IK has moderate predictability for HML with 15% in-sample R^2 and 17% out-of-sample R^2. Another intriguing result is that equipment IK, as a good predictor for aggregate risk premium, predicts risk premia of less risky portfolio deciles (Size10, Value1, Inv10) with higher R^2 and more negative slope coefficients, except for profitability portfolios.

[29] I thank Dimitris Papanikolaou for sharing the data series of the IMC portfolio and Lu Zhang for sharing the data series of q factors.

[30] The correlation between CMA and IA is 0.92; see Zhang (2017) for discussions on the two investment factors.

[31] Equipment IK has a 9% in-sample R^2 for ROE at an 8-quarter horizon (not tabulated) but zero predictability at a 20-quarter horizon (tabulated).

[32] The correlation between RMW and ROE is 0.67.

[33] IMC has a negative correlation with CMA (-0.52) and IA (-0.55).

1.3 Model

To explain the stronger power of equipment investment than structures investment for predicting returns and the lagging behavior of structures investment to total factor productivity (TFP), in this section I build a general equilibrium production model that features a longer TTB for structures than for equipment.

1.3.1 Economic Environment

There is a representative firm and a representative household in the aggregate production economy. The representative firm has a Cobb-Douglas production function F with self-accumulated equipment capital K_{et}, structures capital K_{st} and employed household labor L_t as inputs,

$$Y_t = F(K_{et}, K_{st}, L_t) = A_t K_{et}^{\alpha_e} K_{st}^{\alpha_s} (Z_t L_t)^{1-\alpha_e-\alpha_s},$$

where Y_t is the total output, α_e (α_s) is the production share of equipment (structures), A_t is the TFP, and Z_t is the deterministic growth component. A_t follows an AR(1) process,

$$\log(A_{t+1}) = \rho_a \log(A_t) + \epsilon_{t+1},$$

where ρ_a ($0 < \rho_a < 1$) is the persistence parameter, and ϵ is the TFP shock, which follows a normal distribution, $\epsilon \sim N(0, \sigma_a^2)$. Z_t grows exponentially at a constant rate μ starting from the normalized initial value 1, $Z_t = \exp(\mu t)$.

The firm accumulates structures capital from the undepreciated structures capital left from the previous period and the new structures investment,

$$K_{s,t+1} = (1 - \delta_s)K_{st} + X_{s,t-J_s+1}, \tag{1.3}$$

where δ_s is the depreciation rate of structures, and J_s is the TTB period for structures investment. It takes J_s periods for $X_{s,t-J_s+1}$, the structures investment project initiated at time $t-J_s+1$, to become productive capital. Therefore, there are J_s structures projects each period with $1, 2, ..., J_s$ periods to completion, respectively. Total investment expenditures of structures at time t, denoted as I_{st}, are split into those J_s projects as follows:

$$I_{st} = \sum_{j=1}^{J_s} \omega_j^s X_{s,t-j+1}, \quad \sum_{j=1}^{J_s} \omega_j^s = 1, \tag{1.4}$$

where $X_{s,t-j+1}$ the investment project initiated at time $t-j+1$ with $J_s - j + 1$ periods to completion, and ω_j^s is the fraction of investment cost incurred in the jth stage of the project.[34] $\{\omega_j^s\}_{j=1}^{J_s}$ are structural parameters, time-invariant and project-independent. They sum equal to one and reflect how the investment cost is distributed over the stages of a project. Similarly, the capital accumulation equation and investment equation for equipment are as follows:

$$K_{e,t+1} = (1-\delta_e)K_{et} + X_{e,t-J_e+1}, \tag{1.5}$$

$$I_{et} = \sum_{j=1}^{J_e} \omega_j^e X_{e,t-j+1}, \quad \sum_{j=1}^{J_e} \omega_j^e = 1. \tag{1.6}$$

I assume $J_e < J_s$ to capture that structures require a longer time to build. The standard RBC model, as in Cooley and Prescott (1995), assumes a single type of capital with a one-period TTB. This corresponds to $J = 1$ and $X_t = I_t$.

The firm incurs adjustment costs for adjusting capital stocks,

$$G_i(K_{it}, X_{i,t-J_i+1}) = \frac{\eta_i}{\nu_i}\left(\frac{X_{i,t-J_i+1}}{K_{it}} - \bar{\delta}_i\right)^{\nu_i} K_i, \quad i = e, s,$$

where G_i is the adjustment cost function and is homogeneous degree of one (HD1) with respect to K_i and X_i. η_i and ν_i (capturing curvature) are adjustment cost parameters. $\bar{\delta}_i = e^{\mu} - 1 + \delta_i$ is the growth-adjusted depreciation rate, $i = e$ for equipment and $i = s$ for structures.

The firm is all equity-financed. The residual cash flow, i.e., dividend D_t, is distributed to the equity-holder, i.e., the household, after the firm pays the investment costs $I_{et} + I_{st}$,

[34] I have adopted the simplified notation for investment projects X_{t-j}, as in Christiano and Vigfusson (2003) and Chen (2016). The original Kydland and Prescott (1982) would denote X_{t-j} as $X_{J-j,t-j}$, which keeps track of both the time when the project is initiated $(t-j)$ and periods to completion $(J-j)$. This more complex notation would be more suitable for the recursive formulation of the dynamic programming problem.

the capital adjustment costs $G_e(t)+G_s(t)$, and the wage payments W_tL_t,

$$D_t = Y_t - I_{et} - I_{st} - G_e(K_{et}, X_{e,t-J_e+1}) - G_s(K_{st}, X_{s,t-J_s+1}) - W_tL_t. \tag{1.7}$$

The firm maximizes the cum-dividend firm value V_t (P_t+D_t, P_t is the ex-dividend firm value) using the stochastic discount factor (SDF) M_t implied from the household's optimality conditions,

$$V_t \equiv P_t + D_t = \max_{\{K_{e,t+J_e+j}, X_{e,t+j}, K_{s,t+J_s+j}, X_{s,t+j}, L_{t+j}\}_{j=0}^{\infty}} E_t\left[\sum_{j=0}^{\infty} M_{t,t+j} D_{t+j}\right]$$

subject to the capital accumulation equations (1.3) and (1.5), the investment equations (1.4) and (1.6), and the cash flow constraint (1.7).

The representative household has external habit preferences (see Campbell and Cochrane (1999) and, more recently, Chen (2017)). The household maximizes lifetime utility subject to the budget constraint,

$$\max_{\{C_{t+j}, L_{t+j}, \chi_{t+j+1}, B_{t+j+1}\}_{j=0}^{\infty}} E_t\left[\sum_{j=0}^{\infty} \beta^j \frac{(C_{t+j} - H_{t+j})^{1-\gamma} - 1}{1-\gamma}\right]$$

$$C_t + P_t\chi_{t+1} + B_{t+1} \leq W_tL_t + (P_t + D_t)\chi_t + R_{ft}B_t.$$

β is the time discount factor and γ is the relative risk aversion. At period t, the household consumes C_t, buys χ_{t+1} share of stocks at price P_t and bonds B_{t+1}, and receives income from wage W_tL_t and portfolio holdings, including stock holdings $(P_t + D_t)\chi_t$ and bond holdings $R_{ft}B_t$, where R_{ft} is the gross risk-free interest rate. H is the habit level the household's utility from consumption depends on. Define the aggregate surplus consumption ratio $\hat{S}$ as

$$\hat{S}_t \equiv \frac{\hat{C}_t - H_t}{\hat{C}_t} \qquad \hat{s}_t \equiv \log(\hat{S}_t),$$

where $\hat{x}$ denotes aggregate variable x. $\hat{s}_t$ is assumed to follow,

$$\hat{s}_{t+1} = (1-\rho_s)\bar{s} + \rho_s\hat{s}_t + \lambda_s(\log(\hat{C}_{t+1}) - \log(\hat{C}_t) - \mu).$$

In the endowment economy model of Campbell and Cochrane (1999), λ_s is time-varying and reverse-engineered to achieve a constant risk-free rate. In the production economy here, I follow Chen (2017) and assume that λ_s is constant, $\lambda_s = 1/\bar{S} - 1$. Since there is a representative household, $C_t = \hat{C}_t$ and $S_t = \hat{S}_t$; thus I drop the *hat* henceforth.

In equilibrium, all markets clear. The clearing of the goods market implies the aggregate resource constraint,

$$C_t + I_{et} + I_{st} + G_e(K_{et}, X_{e,t-J_e+1}) + G_s(K_{st}, X_{s,t-J_s+1}) = Y_t.$$

The labor market clears. Since leisure is assumed to not enter the utility function, labor is inelastically supplied at the household's endowment of one unit, $L_t = 1$. The asset markets clear: $\chi_t = 1$ and $B_t = 0$. That is, there is one share of stock and zero net supply of risk-free bonds in the economy.

1.3.2 Investment Q and Asset Prices

Let the Lagrange multipliers on equations (1.5) and (1.3) be q_e and q_s, respectively. The first order condition for X_{it} implies

$$E_t(M_{t,t+J_i-1}q_{i,t+J_i-1}) = E_t\left[M_{t,t+J_i-1}G_{X_i}(K_{i,t+J_i-1}, X_{it})\right] + E_t\left(\sum_{j=1}^{J_i} M_{t,t+j-1}\omega_j^i\right), \quad i = e, s. \tag{1.8}$$

q_e (q_s) is the shadow price or marginal q of equipment (structures) capital. G_{X_i} denotes the partial derivative of function G_i with respect to X_i. The left-hand-side is the marginal benefit of investment in the *new* project X_{it}. Due to TTB, the one additional unit of new investment will become productive capital at time $t+J_i-1$ and can be sold at price $q_{i,t+J_i+1}$. The right-hand-side is the marginal cost. The first term is the adjustment cost that occurs at time $t + J_i - 1$. The second term reflects how the one additional unit of investment in the new project goes into investment expenditures across the stages of the project. Due to TTB, the costs and benefits occur with time lags, to which expectations and discounting are thus applied.[35]

[35] When $J_i = 1$, the standard q-investment equation appears, $q_{it} = G_{X_i}(K_{it}, X_{it}) + 1$.

The first-order condition for $K_{i,t+1}$ implies the asset pricing equation

$$E_t M_{t,t+1} R_{i,t+1} = 1, \quad i = e, s, \tag{1.9}$$

where

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{S_{t+1}}{S_t}\right)^{-\gamma},$$
$$R_{i,t+1} = \frac{F_{K_i}(t+1) - G_{K_i}(K_{i,t+1}, X_{i,t-J_i+2}) + (1-\delta_i) q_{i,t+1}}{q_{it}}.$$

M is the SDF implied from the household's optimality conditions. G_{K_i} denotes the partial derivative of function G_i with respect to K_i. $R_{i,t+1}$ is the investment return in equipment ($i = e$) or structures ($i = s$). Its denominator is the marginal cost of installing an additional unit of capital at time t, q_{it}, and its numerator is the corresponding benefits at time $t+1$, which includes the marginal product of capital $F_{K_i}(t+1)$, the sale value of the undepreciated extra unit of capital $q_{i,t+1}(1-\delta_i)$, and the savings in adjustment cost $-G_{K_i}(K_{i,t+1}, X_{i,t-J_i+2})$.

Define the stock return $R_{m,t+1}$ as the firm's cum-dividend value divided by the previous period ex-dividend value, and the total investment return $R_{I,t+1}$ as the value-weighted return of equipment investment and structures investment,

$$R_{m,t+1} = \frac{P_{t+1} + D_{t+1}}{P_t},$$
$$R_{I,t+1} = \frac{q_{et} K_{e,t+1}}{q_{et} K_{e,t+1} + q_{st} K_{s,t+1}} R_{e,t+1} + \frac{q_{st} K_{s,t+1}}{q_{et} K_{e,t+1} + q_{st} K_{s,t+1}} R_{s,t+1}.$$

Proposition 1. *Because both the Cobb-Douglas production function and adjustment cost functions are homogeneous of degree 1 (HD1), the firm value P_t can be shown to satisfy*

$$E_{t-J_s+2}(M_{t-J_s+2,t} P_t) = \underbrace{E_{t-J_s+2}[(M_{t-J_s+2,t}(q_{et} K_{e,t+1} + q_{st} K_{s,t+1})]}_{\textit{value of productive capital}}$$
$$+ \underbrace{E_{t-J_s+2}\left(\sum_{j=1}^{J_s-1} M_{t-J_s+2,t-J_s+j+1} \omega_j^s X_{s,t-J_s+2}\right) + \ldots + E_{t-J_s+2}\left(M_{t-J_s+2,t} \omega_1^s X_{st}\right)}_{\textit{value of unfinished structures projects}}$$

$$+ \underbrace{E_{t-J_s+2}\left(\sum_{j=1}^{J_e-1} M_{t-J_s+2,t-J_e+j+1}\omega_j^e X_{e,t-J_e+2}\right) + ... + E_{t-J_s+2}\left(M_{t-J_s+2,t}\omega_1^e X_{et}\right)}_{\textbf{\textit{value of unfinished equipment projects}}}. \tag{1.10}$$

Proof. See Appendix A.4 for the derivation. □

The value of the firm equals the value of the productive capital, plus the value of the completed parts of all the unfinished equipment and structures projects.[36] When $J_e = J_s = 1$, the firm value equals the value of the productive capital, and the average q (Q) equals the (capital-weighted) marginal q (Hayashi (1982)),

$$P_t = q_{et}K_{e,t+1} + q_{st}K_{s,t+1}$$
$$\Rightarrow Q \equiv \frac{P_t}{K_{e,t+1} + K_{s,t+1}} = \frac{K_{e,t+1}}{K_{e,t+1} + K_{s,t+1}} q_{et} + \frac{K_{s,t+1}}{K_{e,t+1} + K_{s,t+1}} q_{st}. \tag{1.11}$$

Also, the stock market return equals the investment return, $R_{m,t+1} = R_{I,t+1}$ (Cochrane (1991); Restoy and Rockinger (1994)). When $J_e = J_s = 2$, equation (1.10) can be simplified without expectation,[37]

$$P_t = q_{et}K_{e,t+1} + q_{st}K_{s,t+1} + \omega_1^e X_{et} + \omega_1^s X_{st}.$$

X_{it} is the newly initiated project, which will be completed w_1^i fraction in this period and w_2^i fraction in the next period. The completed w_1^i fraction of the project contributes to the firm value in addition to the productive capital. Due to the existence of unfinished projects, the average q does not equal the marginal q. The stock return can be shown to satisfy

$$R_{m,t+1} = \frac{q_{et}K_{e,t+1}}{P_t} R_{e,t+1} + \frac{q_{st}K_{s,t+1}}{P_t} R_{s,t+1}$$
$$+ \frac{(q_{e,t+1} - G_{X_e}(K_{e,t+1}, X_{et}) + \omega_2^e)X_{et}}{P_t} + \frac{(q_{s,t+1} - G_{X_s}(K_{s,t+1}, X_{st}) + \omega_2^s)X_{st}}{P_t}.$$

[36] See equation (32) in Altuğ (1993), who derives a similar equation under partial equilibrium.

[37] Kuehn (2009) derives the firm value in the case of a single type of capital with two-period TTB. I derive a more general expression for firm value when there are two types of capital with potentially different multiple TTB periods. The expression can easily be extended to the case of multiple (more than two) types of capital.

The stock return does not equal the investment return, $R_{m,t+1} \neq R_{I,t+1}$. The introduction of multiple-period TTB breaks down the equivalence between average q and marginal q and between the stock return and the investment return.

Finally, the risk-free rate is defined as

$$R_{ft} = 1/E_t(M_{t,t+1}).$$

And the risk premium is $R_{ex,t} = R_{mt} - R_{f,t-1}$.

1.4 Quantitative Results

In this section, I first calibrate the model. Then I show that the model matches the empirical moments for macro quantities and asset prices. Next, I demonstrate that the model generates the lagging behavior of structures investment to TFP and the better return predictability for equipment investment than for structure investment, as in the data. I examine the model mechanism through the impulse response functions. After that, I show that the model provides theoretical support for previous empirical findings of return predictability from planned investment. Finally, I show that discount rates drive the variation in the dividend-price ratio in the model, as in the data.

1.4.1 Calibration

The model is calibrated at quarterly frequency. Table 1.10 shows the parameter values. Several are from Chen (2017), including the average GDP per capita growth rate μ set to 0.0048, the persistence of TFP ρ set to 0.98, the time discount factor β set to 0.995, the risk-aversion coefficient γ set to 2 as in Campbell and Cochrane (1999), the persistence of surplus consumption ratio ρ_s set to 0.98, and the steady state of surplus consumption ratio $\bar{S}$ set to 0.07. The volatility of TFP shock σ_a is set to 0.01 to largely match the average volatility of GDP growth of 0.97. It is between the value of 0.007 used in Cooley and Prescott (1995) and 0.018 used in Boldrin et al. (2001).

[Insert Table 1.10 about here]

The rest of the parameters capture the heterogeneities between equipment and structures. First, the growth-adjusted depreciation rates of equipment and structures, $\bar{\delta}_e$ and $\bar{\delta}_s$, are set to be the average quarterly equipment and structures investment rates 0.0386 and 0.0125, which results in depreciation rates 0.0338 and 0.0077. Second, the capital share $\alpha_e + \alpha_s$ is set to 0.36 as in Tuzel (2010). Individual production shares for equipment and structures, α_e and α_s, are then calibrated to match the average relative ratio of private nonresidential equipment investment to structures investment, 1.86. This gives α_e as 0.202 and α_s as 0.158, which are close to the values of 0.216 and 0.144 used by Tuzel (2010). The resulting steady state of the relative ratio of equipment capital stock to structures capital stock is about 0.6, which is consistent with Tuzel (2010) and Jermann (2010).

The third heterogeneity is the capital adjustment cost. The literature is not settled on whether equipment or structures is more costly to adjust. Israelsen (2010) estimates higher adjustment costs for equipment, while Tuzel (2010) and Jermann (2010) calibrate higher adjustment costs for structures.[38] Since the adjustment costs in the model are zero at the deterministic steady state, there is no counterpart in macro data that can be used to calibrate the adjustment cost parameters. I follow Greenwood et al. (2000) and set the same parameter values for equipment and structures. I use the standard quadratic adjustment cost, $\nu_e = \nu_s = 2$. I calibrate the adjustment cost parameter η to largely match the relative volatility of equipment and structures investment growth to output growth, 3.65 and 3.12, respectively. This leads to $\eta_e = \eta_s = 50$.[39]

Finally, the TTB specifications are different for equipment and structures. As the evidence presented in Section 1.2.3 shows, the TTB for equipment J_e is set to 1 to capture a 1-quarter equipment delivery lag, and the TTB for structures J_s is set to 5 to capture the long planning and construction lags. Since $J_e = 1$, $\omega_e = 1$. The project completion pattern parameters for structures (ω_s =(0.10,0.15,0.20,0.25,0.30)) are set to capture the idea of time-to-plan in Christiano and Todd (1996) and to match the pattern of the increasing cross-correlations between structures investment growth and TFP growth.

[38] The adjustment costs in Israelsen and Jermann's models are non-quadratic and for aggregate capital adjustment, while Tuzel's adjustment cost is quadratic and asymmetric and for firm-level adjustment.

[39] The numbers seem high, but Chen (2017) shows that an adjustment cost of 100 results in less than 1% mean adjustment cost as a percentage of output. The adjustment cost percentages in my calibration are 0.09%, 0.17%, and 0.26% for equipment, structures, and total capital, respectively.

1.4.2 Model Statistics

Because there are seven state variables in the model, namely $\{K_{et}, K_{st}, \{X_{s,t-i}\}_{i=1}^{4}, A_t\}$, global solution methods are generally infeasible. I solve the model using the perturbation method with Dynare++ third-order approximation. I first normalize the model variables by dividing by the deterministic growth component Z_t and solve the model in terms of the stationary variables. Then I add Z_t back into the variables in the simulations. The model is simulated 500 times each 280 quarters, and mean statistics are reported.

Table 1.11 reports the statistics for macro quantities and asset prices across various model variants. The benchmark model matches the volatilities and correlations of the macro quantities well. Consumption is less volatile than output, while investment fluctuates much more than output. As for asset prices, the model generates a high and volatile risk premium (4.28% mean and 15.01% volatility), as in the data.[40] However, the model overshoots the mean and volatility of the risk-free rate in comparison with the data: 1.92% versus 0.57% for the mean, and 5.84% versus 2.52% for the volatility.[41]

[Insert Table 1.11 about here]

To investigate how each heterogeneity between equipment and structures (depreciation, production share, or TTB) affects model predictions, I strip each heterogeneity out of the model benchmark separately in three alternative model scenarios, *Models 1-3*, whose model statistics are shown in Table 1.11. I find that removing heterogeneity in the depreciation rate or production share has relatively small effect on model fits. Removing TTB, however, dramatically reduces model fit in both macro quantities and asset prices.

In *Model 1*, in which equipment and structures have the same (growth-adjusted) depreciation rate, $\bar{\delta}_e = \bar{\delta}_s = 0.025$, the volatility of equipment investment increases from 3.84% to 4.52%, while the volatility of structures investment decreases from 3.14% to 1.49%. Equipment at a lower depreciation rate needs a larger adjustment when responding to the same

[40] I assumed that the firm is all equity-financed with zero leverage. Assuming a debt-equity ratio of 0.5 instead will bring the mean risk premium up to a closer match at 6.42%, but overshoot the volatility of the risk premium at 22.51%.

[41] The mean and volatility of the risk-free rate in the data could be higher if we use a longer sample. For example, Campbell (2003) reports that the mean and volatility of the risk-free rate are 2.02% and 8.81%, respectively, over the sample of 1891-1998.

amount of a productivity shock, which leads to higher volatility. The opposite is true for structures. In addition, the stock return and risk premium fall, because the reduced risk due to higher depreciation of structures outweighs the added risk due to the lower depreciation of equipment.

In *Model 2*, in which equipment and structures have the same production share, $\alpha_e = \alpha_s = 0.18$, a lower production share of equipment raises the volatility of equipment investment from 3.84% to 4.21% and leads to more risky equipment investment. The opposite is true for structures; structures investment becomes less volatile and less risky. Because the added risk of equipment investment exceeds the reduced risk of structures investment, the means and volatilities of stock return and risk premium all rise.

When both equipment and structures have only a 1-quarter TTB in *Model 3* (no TTB, $J_e = J_s = 1$), as in the standard RBC model, the short-run supply of structures capital becomes elastic. The volatility of structures investment rises from 3.14% to 5.77%, and the volatility of equipment investment falls from 3.84% to 1.67%. This suggests that a longer TTB reduces the elasticity of structures capital supply and dampens the volatility of structures investment. This also explains why I do not need to calibrate a higher adjustment cost for structures as in Tuzel (2010) (her model has the standard one-period TTB) to match the volatilities of structures investment and equipment investment. Because the supply of overall capital is more elastic in the economy, consumption absorbs less TFP shock and becomes less volatile; its volatility decreases from 0.50% to 0.36%. In addition, the correlation between output and structures investment becomes too high, at 0.97, in comparison with the data at 0.34. As for asset prices, the means and volatilities of the stock return and risk premium all fall, due to the higher elasticity of capital supply. Also, the mean of the risk-free rate rises from 1.92% to 3.81% and its volatility decreases from 5.84% to 0.54%, because the TFP shock loads less in consumption.

1.4.3 Model Discussion

In addition to TTB, several other important elements are built into the model, such as capital adjustment cost, habit, and TTP. Both TTB and capital adjustment cost reduce the elasticity of capital supply. TTB makes only the short-run capital supply inelastic, while the capital adjustment cost reduces the elasticity in both the short run and long run. Ceteris paribus, a lower elasticity of capital supply makes the equilibrium price of capital more volatile (see Kogan and Papanikolaou (2012), Figure 1, for a graphic illustration). On the other hand, habit preference induces strong motives in consumption smoothing and amplifies fluctuations in capital demand. This magnifies the effect of the low elasticity of capital supply due to TTB and capital adjustment cost, and boosts the size and volatility of the risk premium. In addition, TTP makes investment more risky by loading investment expenditures more on past investment decisions. I discuss in *Models 4-8* how these various model elements are necessary to achieve reasonable macro quantities and asset prices.

When there is no TTP in *Model 4* ($\omega_i^s = 0.2, i = 1, ..., 5$), structures investment comoves with output more. Both the volatilities of structures investment and equipment investment decrease slightly, while the volatility of aggregate investment increases a bit. This leads to slightly lower consumption volatility. The removal of TTP decreases the stock return and the risk premium.

When there is no habit or utility is CRRA (constant relative risk aversion) in *Model 5* ($H_t = 0$), consumption volatility jumps to 1.12% and investment volatilities fall. The removal of habit weakens the desire to smooth consumption and reduces the elasticity of capital demand, resulting in a high average risk-free rate at 5.77%, a small risk-free rate volatility at 0.48%, a small stock return volatility at 3.35%, and a low average risk premium at 0.17%. As is evident from *Models 5-8*, habit is necessary to generate a sizable and volatile risk premium.

When adjustment cost is further removed, in addition to habit preference, in *Model 6* ($H_t = 0, \eta_e = \eta_s = 0$), both equipment and structures investment become more volatile. Structures investment has a negative correlation to output at -0.21. This means that structures investment decreases on impact in response to a positive TFP shock, which translates into a small negative risk premium at -0.02%. In comparison to *Model 5*, the zero adjustment cost in *Model 6* increases the elasticity of the capital supply at both short and long horizons. As a result, the volatilities of the stock return and the risk premium decline from 3.35% and 3.26%, respectively, in *Model 5* to 0.23% and 0.08% in *Model 6*.

In *Model 7*, TTB instead of the adjustment cost is removed, in addition to habit preference ($H_t = 0, J_e = J_s = 1$). Relative to *Model 5* (no habit alone), the rise in investment volatility is smaller than that in *Model 6* (no habit and no adjustment cost). Also, the decline in stock return volatility is smaller. This is because TTB reduces only the short-run elasticity of structures capital supply, but adjustment cost technology has a long-lasting effect on the capital adjustment of both equipment and structures. In addition, both equipment investment and structures investment show perfect correlations with output.

In *Model 8*, in which there is no habit, no TTB, and no adjustment cost ($H_t = 0, J_e = J_s = 1, \eta_e = \eta_s = 0$), both equipment investment and structures investment become highly volatile. Because they move in opposite directions (as seen from the positive correlation between output and equipment investment but the opposite for structures investment), the volatility of aggregate investment is reasonable at 2.45%. Since the capital supply becomes perfectly elastic without TTB and adjustment cost, the risk premium is negligible and returns are not volatile.

1.4.4 Predictions on Cross-Correlations

The benchmark model generates similar investment-TFP correlations as in the data. Figure 1.4 depicts how investment growth correlates with TFP growth in the model. The model generates comovement between equipment investment and TFP and a 4-quarter lag of structures investment relative to TFP, as in the data (Figure 1.2). However, the mildly negative correlations for lags at 1-4 quarters between equipment and TFP is inconsistent with the data. In addition, the model produces higher investment-TFP correlations than in the data. One possible reason is that I ignore other components of investment in the model, including inventory, land, and IPP, which are used in John Fernald's TFP data series.

[Insert Figure 1.4 about here]

To investigate which model assumption of capital heterogeneity leads to the difference in TFP correlations between equipment and structures, Figure 1.4 also shows the investment-TFP cross-correlations for three alternative models, stripping out each heterogeneity separately, in which equipment and structures have the same depreciation rate (*Model 1*), the same production share (*Model 2*), and the same 1-quarter TTB (*Model 3*). The cross-correlations in *Model 1* and *Model 2* are similar, as in the benchmark model. But both equipment investment and structures investment comove with TFP when longer TTB for structures is assumed away in *Model 3*. The results suggest that the heterogeneity in TTB is the key driver of the lagging behavior of structures investment.

1.4.5 Predictions on Return Predictability

The benchmark model also generates reasonable results in return predictability from investment rates as in the data. Table 1.12 reports the in-sample R^2 and regression slopes b for predictive predictions for the stock return, risk premium, and risk-free rate across various horizons ranging from 1 quarter to 20 quarters.[42]

[Insert Table 1.12 about here]

In predicting the stock return, the model produces higher R^2 for equipment IK than structures IK at both short horizons and long horizons, as in the data. The R^2 at a 1-quarter horizon and 20-quarter horizon for equipment versus structures are 7.2% versus 0.8% and 27.9% versus 15.2%. Equipment IK over-predicts the stock return at the short horizon (20.9% R^2 in the model versus 7.9% R^2 in the data at a 4-quarter horizon), while structures IK over predicts the stock return at the long horizon (15.2% R^2 in the model versus 3.7% R^2 in the data at a 20-quarter horizon). This result suggests that there may be a longer TTB for equipment—and an even longer TTB for structures—than what the model assumes. In addition, the predicting slopes are negative, as in the data. When discount rates fall, investment rises.

As for predicting the risk premium, the model generates similar R^2 for structures IK, as in the data but relatively low R^2 for equipment in comparison with the data. The reason is that equipment IK predicts the risk-free return negatively with large R^2 at short

[42] Note that model-implied investment rates are generated using the simulated investment data and the perpetual inventory method, which is how investment rates in the data are constructed. Because the model assumes multiple-quarter TTB for structures—but a 1-quarter TTB is assumed in the data—directly dividing the simulated structures investment by the simulated structures capital stock is not consistent with the data procedure.

horizons. Because the risk premium is the difference between the stock return and the risk-free return, the combination of negative predictions for both the stock return and risk-free return results in less negative predicting slopes and lower R^2 for predicting the risk premium than for predicting the stock return.

To investigate which model assumption of capital heterogeneity drives the difference in return predictability between equipment and structures, Table 1.12 also shows predictability results for the three alternative models, stripping out each heterogeneity separately, in which both equipment and structures have the same depreciation rate (*Model 1*), the same production share (*Model 2*), and the same 1-quarter TTB (*Model 3*).

In *Model 1* and *Model 2*, the better performance of equipment is preserved. But in *Model 3*, there is no significant prediction difference between equipment and structures. This is because when both equipment investment and structures investment react to productivity shocks in the same way, their marginal q's contain the same set of information reflected in stock prices. Since marginal q is a linear function of IK due to the assumption of the quadratic adjustment cost, equipment IK and structures IK have similar information for predicting returns. The results across the three alternative models imply that the assumption of the longer TTB for structures is the driver of the difference in return predictability.

1.4.6 Model Mechanism

To examine the model mechanism, Figure 1.5 depicts the impulse responses of model variables to a positive one standard deviation of TFP shock (1%) at time 1 across three models, namely, the benchmark model, *Model 3* when there is only a 1-quarter TTB, and *Model 6* when utility is CRRA and adjustment cost is zero.

When a positive TFP shock hits the economy in the benchmark model, output, consumption, and equipment investment rise on impact. Structures investment also rises on impact, but it takes 5 quarters for it to achieve the maximum response due to TTB and TTP. Because the stock return rises on impact and then declines, the delayed response of structures investment renders it less informative than equipment investment for predicting the stock return. The structures investment decision (X_s) shows responses similar to those for equipment investment, and is much more volatile than the structures investment (expenditures).[43] In addition, because output rises on impact, while structures investment increases a little, equipment investment and consumption have to overshoot to absorb the productivity shock. Thus equipment investment and consumption gradually decline from quarter 1 to quarter 5, when the supply of structures capital becomes elastic.

[Insert Figure 1.5 about here]

Because consumption overshoots on impact, the risk-free rate decreases in the short run. To see this, first, the surplus-consumption ratio (not shown in the figure) shares the same pattern of impulse response as consumption. So does the consumption surplus, which is consumption multiplied by the surplus-consumption ratio ($C_t - H_t = C_t * ((C_t - H_t)/C_t)$). Because the consumption surplus rises on impact, the marginal utility of current consumption surplus falls. The marginal utility of future short-run consumption surplus also falls, but by a lesser amount, because the consumption surplus declines in the short run but is still above the stochastic steady state. And the risk-free rate is the ratio of the former marginal utility to the latter marginal utility (up to the multiplication of the time discount factor).

The decline of the risk-free rate in the short run is shared by other models that feature short-run factor inflexibilities, such as the two-sector model with labor and capital immobilities across sectors in Boldrin et al. (2001) and the 1-period TTP model analyzed in that paper as well. This is both a blessing and a curse. The blessing is that the model generates the "inverted leading-indicator property of interest rates" as in the data highlighted in Boldrin et al. (2001): High interest rates today are associated with lower future output.[44] The curse is that the risk-free rate becomes too volatile. Also, the equipment IK will be strongly negatively associated with the short-run risk-free return, because equipment investment rises on impact, while the risk-free rate drops on impact. This weakens the negative predictions of equipment IK for the risk premium, as shown in Table 1.12 above.

When both equipment and structures have a 1-quarter TTB (the "No TTB" case in Figure 1.5), both equipment and structures investment rise on impact. The simultaneous

[43] Similar to equipment investment, X_s predicts stock returns well, as will be shown in Section 1.4.7.

[44] See also Beaudry and Guay (1996) and King and Watson (1996). The standard RBC model generates positive comovement between interest rates and output because the impulse response of consumption is hump-shaped.

movement of equipment investment and structures investment makes both investment rates similarly informative for return fluctuations as shown in Table 1.12. Structures capital becomes elastic in the short run and absorbs part of the productivity shock, which is loaded on consumption and equipment investment before. Therefore, structures investment rises more on impact and becomes more volatile, while equipment and consumption rise less on impact and become less volatile. Consumption does not overshoot, and the risk-free rate has a small volatility. The stock return and risk premium rise on impact, then decline to their stochastic steady states. All of the impulse responses converge to the ones in the benchmark model after the TTB periods for structures, when structures capital becomes elastic in the benchmark model.

When the TTB assumption for structures is retained but habit preference and adjustment cost are removed from the model (the "No Habit No Adj" case in Figure 1.5), the marginal q for equipment equals one and the marginal q for structures investment is smaller than 1 due to TTB and the discounting. It is more beneficial to invest in equipment in the short run. Thus, equipment investment overshoots and structures investment even decreases on impact. Consumption increases on impact and has a hump-shaped response, as in a standard RBC model. As a result, the risk-free rate rises on impact and has a small volatility. Because the removal of habit preference reduces the fluctuation in capital demand and the removal of adjustment cost makes capital supply more elastic, the resulting stock return and risk premium have little volatility.

As noted in Rouwenhorst (1991), the impulse responses oscillate for a TTB model with a single type of capital and no adjustment costs. This is inconsistent with the empirical evidence. Kuehn (2009) shows that adding the *investment* adjustment cost can render the impulse responses to become smooth but adding *capital* adjustment cost does not work. Here, even though there is no adjustment cost, the impulse responses are smooth due to the assumption of two types of capital. Equipment has the standard 1-quarter TTB and can absorb the shock upfront. The supply of overall capital (equipment plus structures) is elastic in the short run, although the supply of structures capital is not.

1.4.7 Planned Investment and Return Predictability

The model provides theoretical support for previous empirical findings of return predictability from planned investment, as in Lamont (2000) and Jones and Tuzel (2013b). Table 1.13 shows how the structures investment decision or planned structures investment in the language of Lamont (2000) predicts market returns. The growth rate of planned structures investment ($\log(X_{st}/X_{s,t-1})$) negatively predicts annual market returns with a 10% R^2. The structures investment rate ($X_{st}/K_{s,t+4}$) also negatively predicts annual market returns with 12% R^2. These two results are empirically shown by Lamont (2000) (in his Tables III and V, respectively). One difference is that Lamont's planned investment includes both structures and equipment.[45]

[Insert Table 1.13 about here]

The ratio of structures investment decision to structures investment expenditures (X_{st}/I_{st}) is similar to Jones and Tuzel (2013b)'s ratio of nonresidential building starts to structures investment expenditures (Starts/SI) constructed using the same logic as their new orders to shipment ratio. X_{st}/I_{st} shows the highest predicting R^2 of 25% for annual market returns. The R^2 first increases with the predicting horizon up to 4 quarters, then declines. The pattern is the same when $\log(X_{st}/X_{s,t-1})$ is the predictor; but the R^2 for X_{st}/I_{st} is quantitatively larger than the R^2 for $\log(X_{st}/X_{s,t-1})$.

The pattern of R^2 for Starts/SI is different from X_{st}/I_{st}. It increases with the predicting horizon, and is small at short horizons and large at long horizons. This could be due to the inclusion of government structures investment in Starts/SI. First, government construction projects usually have longer TTB than private nonresidential construction projects; for example, Census Bureau (1992) reports that the average number of months from start to completion for state and local construction projects is 20.3 months, while the analog for private nonresidential is 14 months. This could lead to predictability's showing up only in long horizons.

[45] Another difference is that for the investment rate that Lamont uses, the capital stock from BEA constructed under the assumption of a 1-quarter TTB, while the structures capital stock in the model has a 5-quarter TTB. Because the structures capital stock is persistent in the model, using instead the capital stock accumulated from the simulated structures investment under the assumption of 1-quarter TTB has little effect on the result.

Second, government investment is negatively correlated with private investment (-0.23 correlation), and positively predicts aggregate risk premium, as shown by Belo and Yu (2013).[46] The decomposition of government investment into equipment and structures shows that the equipment investment rate predicts the risk premium positively at all horizons, while the structures investment rate predicts the risk premium negatively at long horizons in Jones and Tuzel (2013b)'s sample from 1958 to 2009, as shown in Table A.2.[47] If the prediction result for *investment expenditures* in government structures also holds for the *planned investment*, the negative prediction from *government* structures investment at long horizons could reinforce the negative prediction from *private* structures investment, and lead to the large R^2 in long horizons for Starts/SI. It is possible that at short horizons, the negative correlation between government structures investment and private structures investment counteracts the negative prediction of private structures investment for the risk premium and leads to the small R^2 for Starts/SI.

1.4.8 Discount Rates versus Cash Flows

If the stock price increases today, either the expected dividend growth increases or the discount rate falls, or both. Campbell and Shiller (1988) decompose the aggregate dividend-price ratio into long-run stock returns (discount rates) and long-run dividend growth (cash flows), and find that discount rates drive the variation in the dividend-price ratio.[48] In the model, TFP shocks drive variations of both discount rates and cash flows. It is not certain that the return predictability in the model from investment rates truly comes from discount rate variations; it is possible that cash flows drive the variation in the dividend-price ratio and correlate negatively with discount rates. High investment today that predicts lower future discount rates is simply a manifestation for predicting higher future cash flow growth.

This is not the case, however, seen from the impulse responses in Figure 1.5. When a positive TFP shock hits the economy, the stock price (P) and stock return (R_m) rise, while

[46] Relatedly, Bansal et al. (2016) show that there is reallocation from private investment to government investment when productivity uncertainty is high.

[47] The definition of government investment here is slightly different from that of Belo and Yu (2013), who exclude federal defense spending from gross government investment.

[48] See Cochrane (2011) for a recent review.

dividend (D) falls. This suggests that a *positive* TFP shock acts as a *negative* discount rate shock: The stock price has to fall to accommodate the decline in dividends. To verify this formally, I use VAR analysis and perform Campbell-Shiller decomposition for the dividend-price ratio; the results are shown in Table 1.14. It is evident that discount rates truly drive the variation in the dividend-price ratio in the model. Therefore, high investment rates today are indeed predicting lower discount rates.

[Insert Table 1.14 about here]

The model shows regression results similar to those for data for the first-order VAR: The dividend-price ratio predicts the next-year stock return significantly positively but does not predict the next-year dividend growth. The prediction sign in data for dividend growth is positive. The high long-run coefficient for returns and low coefficient for dividend growth suggest that discount rates drive the variation in the dividend-price ratio. The variance decomposition further confirms this; almost all the variation in the dividend-price ratio comes from the variation in discount rates. The discount rates variation as a percentage of total dividend-price variation is over 100% (104.46% in the model and 161.67% in the data), due to the positive correlation between discount rates and cash flows. The variance in discount rates (0.1307) in the model is smaller than that in the data (0.2435), because the stock return in the model has a slightly smaller mean and standard deviation than in the data.

1.5 Conclusion

This paper establishes a new and robust empirical finding: Equipment investment is more tightly linked to stock returns than structures investment. I build a general equilibrium production model with heterogeneous time-to-build for equipment and structures to explain this empirical finding. Equipment investment requires less time to transform into productive capital, and thus it reacts to productivity shocks more promptly than structures investment, and reflects more of the information contained in stock prices. For future research, it will be interesting to explore the implication of heterogeneous TTB for stock returns at firm level.

Although US Compustat does not provide the split of capital investments into equipment and structures, the confidential micro data from the US Census Bureau does have this information, at least for recent sample years. In addition, the international data for some countries, such as the panel of Italian firms studied by Boca et al. (2008), also contain detailed information on investment in equipment and structures.

Figure 1.1: Quarterly Investment Rates

This figure shows the investment-capital ratios of nonresidential total (excluding intellectual property and products), nonresidential equipment, and nonresidential structures over NIPA sample 1947 Quarter 1 to 2015 Quarter 4. Investment data are from NIPA. Capital stocks are constructed with the perpetual inventory method. Shaded areas are NBER-indicated recessions.

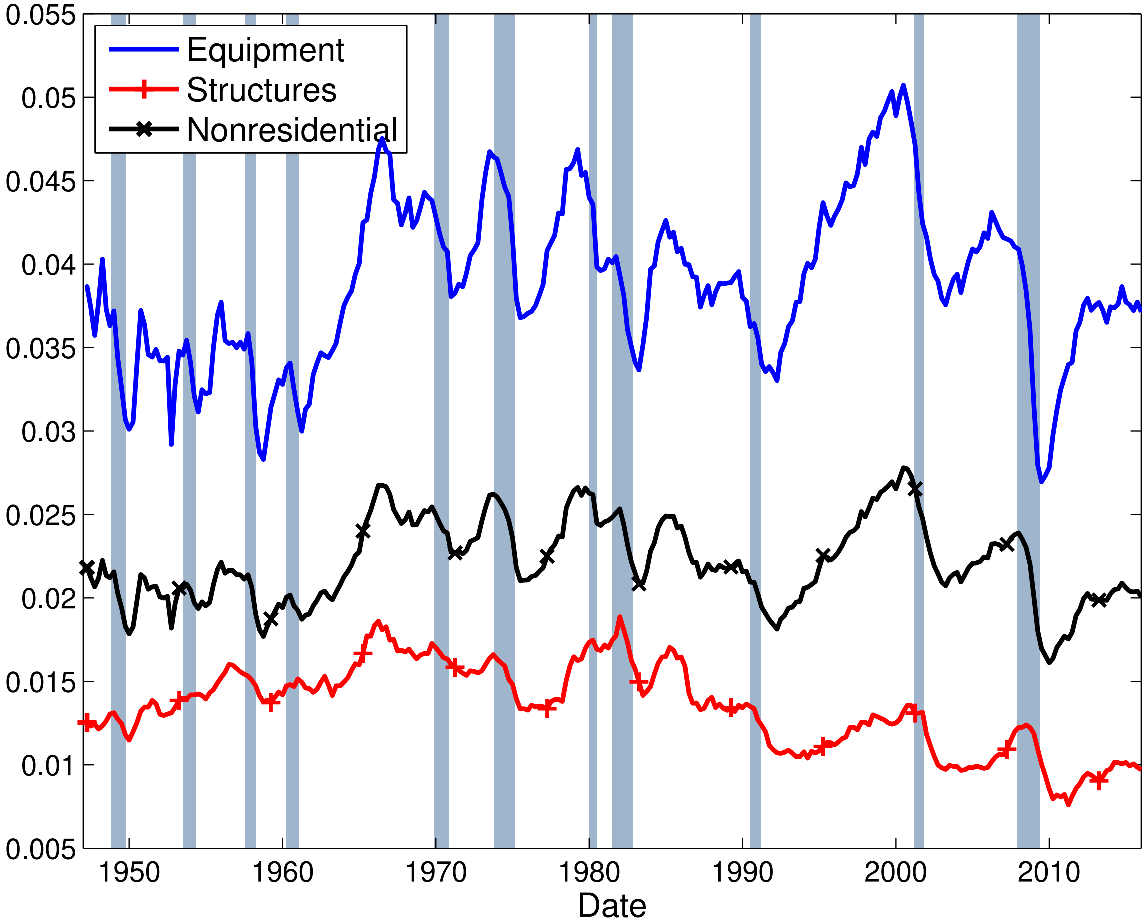


Figure 1.2: Quarterly Cross-Correlations between Investment Growth Rates and TFP Growth Rate

This figure shows quarterly lead-lag correlations between nonresidential investment growth rates (in log) at $t+i$ and TFP growth rate (in log) at t over NIPA sample 1947 Quarter 1 to 2015 Quarter 4. Investment data are from NIPA. *Nonresidential* investment excludes intellectual property and products. *Equipment* is nonresidential equipment. *Structures* is nonresidential structures. TFP data are from John Fernald's website.

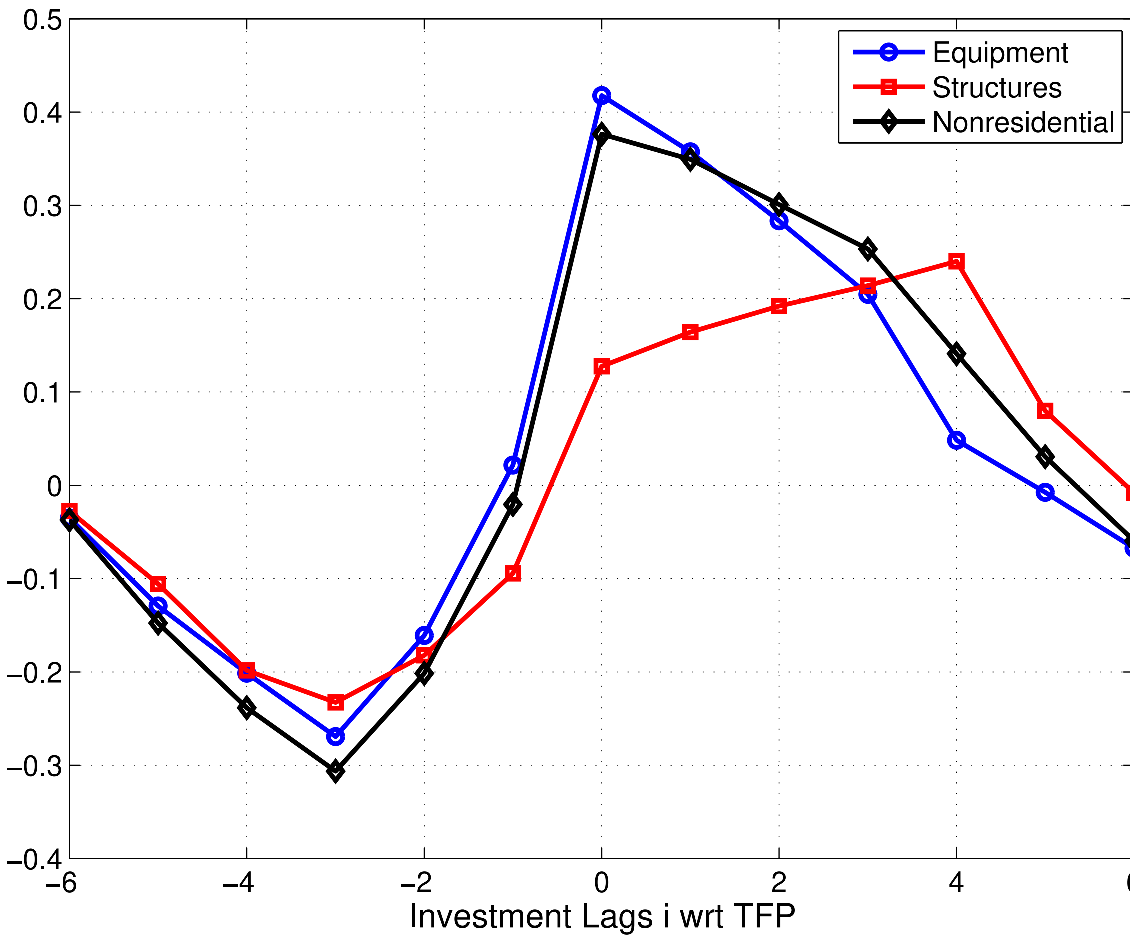


Figure 1.3: Actual and Predicted 5-year Risk Premium

This figure shows the actual and predicted 5-year-ahead risk premium from 1947 Quarter 2 to 2011 Quarter 1. The predictor is equipment investment rate. "IS" means in sample. "OOS" means out of sample. The out-of-sample procedure uses the first half of the sample as the training period, and recursively predicts and retrains in subsequent periods. Shaded areas are NBER-indicated recessions.

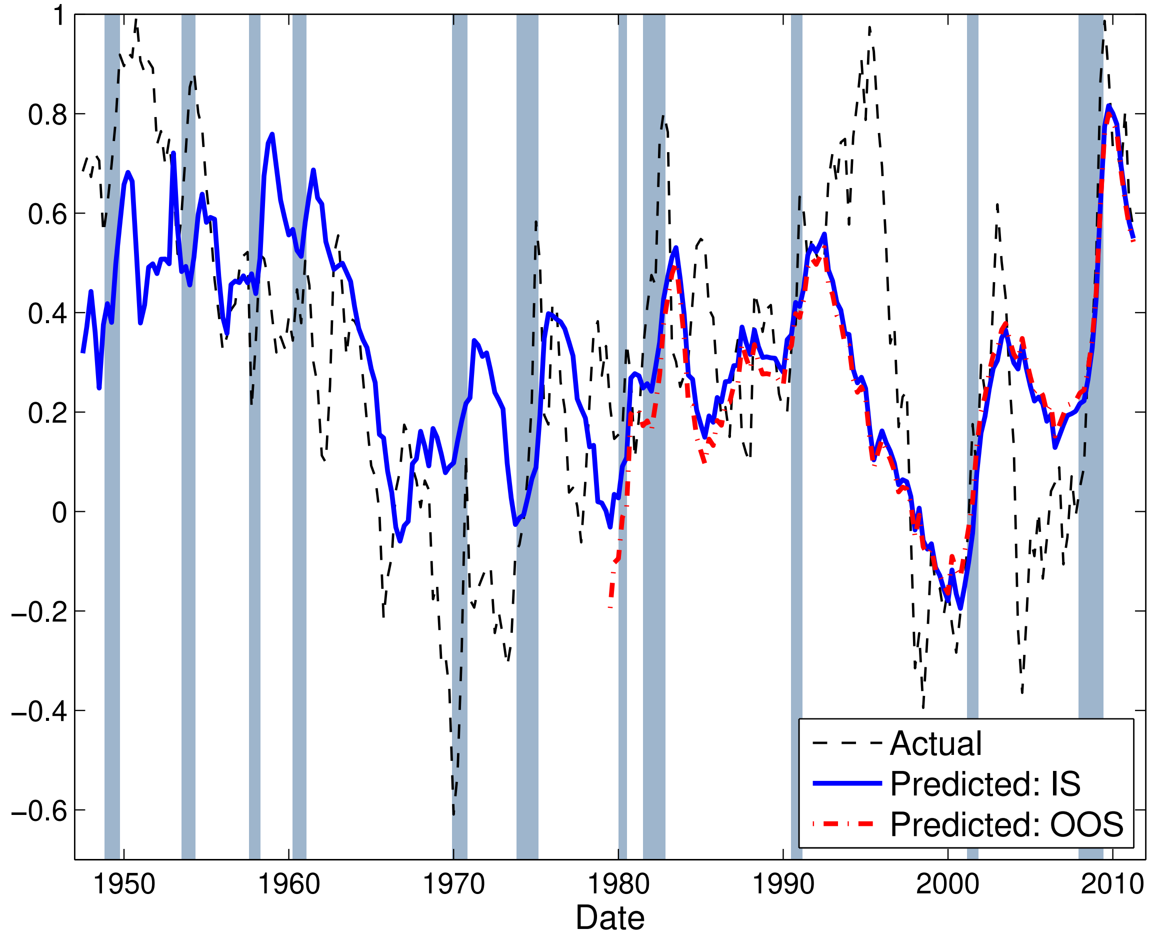


Figure 1.4: Model-Implied Investment and TFP Cross-Correlations

This figure shows the model-implied quarterly lead-lag correlations between investment growth rates (in log) at $t+i$ and TFP growth rate (in log) at t. The model scenarios include Benchmark Model, Model 1 (same depreciation, $\bar{\delta}_e = \bar{\delta}_s = 0.025$), Model 2 (same production share, $\alpha_e = \alpha_s = 0.18$), and Model 3 (no TTB, $J_e = J_s = 1$). Each model is simulated 500 times and the mean correlations are reported.

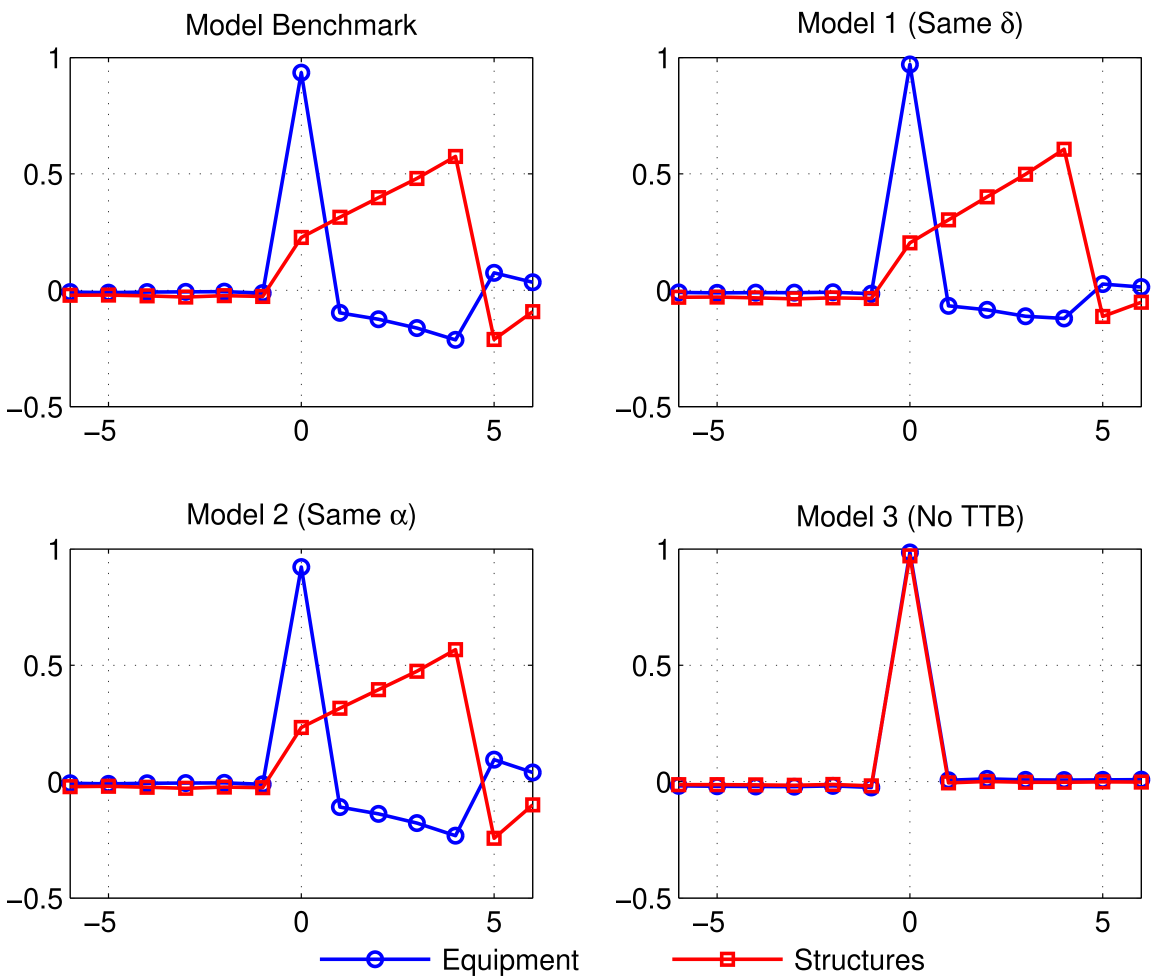


Figure 1.5: Model Impulse Responses to TFP Shocks

This figure shows log deviations of model variables from stochastic steady states in response to a one standard deviation TFP shock at time 1. All plotted responses are scaled by the standard deviation of the TFP shock (1%). Model scenarios include the Benchmark Model, Model 3 (no TTB, $J_e = J_s = 1$), and Model 6 (no habit, no adjustment cost, $H_t = 0, \eta_e = \eta_s = 0$).

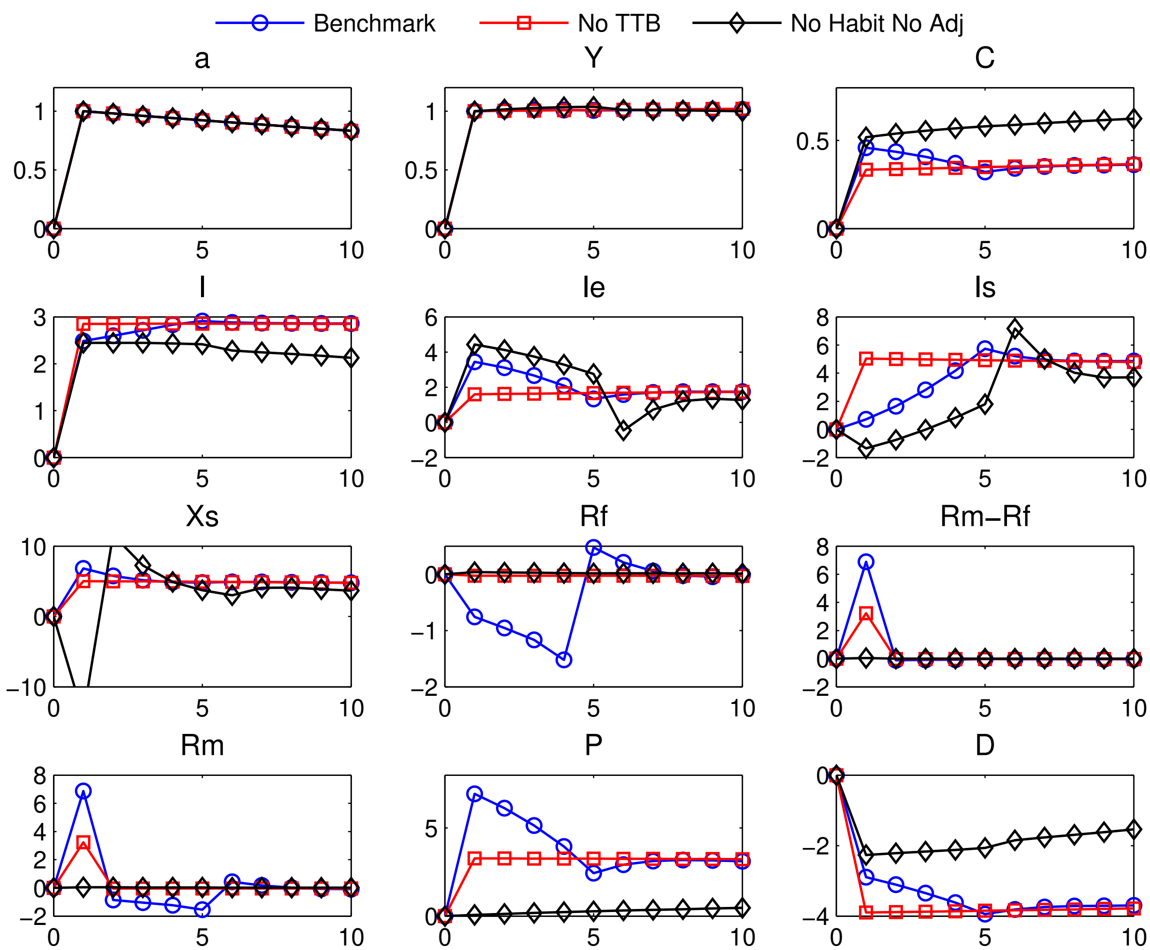


Table 1.1: Descriptive Statistics for Investment Rates

This table reports the descriptive statistics (mean (in percent), standard deviation (Std, in percent), autocorrelation (AC(1)), and correlations) for US quarterly equipment and structures investment rates at aggregate level, asset level, and industry level. Depreciation rates (Dep) for corresponding capital types are also reported. The sample period is 1947Q1-2015Q4 for quarterly aggregate and equipment-asset investment rates, 1959Q1-2015Q4 for quarterly structures-asset investment rates, and 1947-2015 for annual industry investment rates.

Investment Rate (IK)	Dep	Mean	Std	AC(1)	Correlation with Aggregate		
					Nonresi.	Equip.	Struct.
Panel A: Quarterly Aggregate Investment Rates							
Nonresidential	1.26	2.21	0.25	0.971	1.00	0.93	0.56
Equipment	2.72	3.88	0.49	0.965	0.93	1.00	0.26
Structures	0.79	1.35	0.25	0.988	0.56	0.26	1.00
Panel B: Quarterly Asset-Level Investment Rates							
Equipment:							
Information processing	3.11	5.69	0.95	0.969	0.87	0.81	0.52
Industrial	2.40	2.89	0.39	0.957	0.81	0.79	0.54
Transportation	3.28	4.05	0.73	0.923	0.65	0.74	0.25
Other	3.80	4.48	0.50	0.921	0.67	0.72	0.23
Structures:							
Commercial and health care	0.64	1.34	0.39	0.988	0.59	0.35	0.88
Manufacturing	0.82	1.17	0.34	0.970	0.52	0.34	0.78
Power and communication	0.58	1.01	0.20	0.959	0.33	0.14	0.52
Mining exploration, shafts, & wells	1.91	2.15	0.69	0.952	0.21	0.03	0.31
Other structures	0.60	0.96	0.17	0.965	0.56	0.44	0.56

Table 1.1 Continued

Investment Rate (IK)	Dep	Mean	Std	AC(1)	Correlation with Aggregate		
					Nonresi.	Equip.	Struct.
Panel C: Annual Industry-Level Investment Rates							
Equipment:							
Agriculture	14.39	15.92	3.44	0.851	0.12	0.28	-0.27
Mining	14.46	17.92	4.79	0.794	0.19	0.08	0.24
Construction	16.34	18.80	5.35	0.792	0.52	0.71	-0.14
Manufacturing	9.81	12.78	2.12	0.809	0.79	0.80	0.40
Wholesale	14.84	20.54	4.90	0.788	0.64	0.64	0.46
Retail	13.11	18.96	3.06	0.725	0.67	0.71	0.19
Transp & warehousing	8.93	11.27	2.48	0.730	0.67	0.81	0.01
Information	11.90	18.69	2.80	0.657	0.62	0.63	0.22
Profes, scient & techn serv	12.31	22.15	5.70	0.881	0.54	0.60	-0.08
Admin & waste manag serv	12.88	21.14	3.80	0.683	0.54	0.54	0.19
Health care & social assist	15.62	22.81	2.24	0.648	0.35	0.27	0.32
Arts, entert & recreation	14.53	18.81	4.18	0.845	0.33	0.49	-0.16
Accomodation & food serv	14.79	17.77	1.85	0.548	0.58	0.53	0.44
Other serv, except govern	12.96	17.89	4.00	0.783	0.34	0.29	0.23
Structures:							
Agriculture	2.49	2.17	0.83	0.913	0.23	0.19	0.18
Mining	7.01	9.13	2.80	0.882	0.06	-0.21	0.49
Construction	2.75	7.96	5.05	0.886	0.17	-0.03	0.68
Manufacturing	3.22	4.36	1.40	0.853	0.55	0.39	0.76
Wholesale	2.63	7.93	3.60	0.790	0.24	-0.05	0.80
Retail	2.70	6.01	1.89	0.893	0.41	0.21	0.80
Transp & warehousing	2.23	2.98	0.75	0.837	0.15	0.24	-0.41
Information	2.58	5.66	1.45	0.882	0.71	0.49	0.75
Profes, scient & techn serv	2.70	9.06	3.73	0.852	0.11	-0.16	0.65
Admin & waste manag serv	2.48	6.33	2.88	0.899	0.26	0.00	0.74
Health care & social assist	2.18	7.59	3.58	0.940	-0.09	-0.31	0.63
Arts, entert & recreation	3.00	6.76	2.46	0.854	0.17	0.13	0.15
Accomodation & food serv	2.90	6.54	2.71	0.892	0.20	-0.01	0.69
Other serv, except govern	2.23	4.82	2.05	0.954	-0.03	-0.25	0.55

Table 1.2: Cross-Correlations between Investment and TFP, GDP

This table reports the quarterly cross-correlations between aggregate nonresidential investment (equipment and structures) and TFP, and between aggregate nonresidential investment and GDP, $corr(I_{t+i}, X_t)$, where X is TFP or GDP and i is investment lag. Panels A, B, and C report the correlations using first-differenced data, HP-filtered data ($\lambda = 1600$), and bandpass-filtered data (fluctuations from 6 to 32 quarters), respectively. Data for investment and GDP are from NIPA. Data for TFP are from John Fernald's website. The sample period is 1947Q1-2015Q4.

Series	Cross correlations between investment at $t+i$ and TFP or GDP at t												
	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
Panel A: First-Differenced Data													
Equipment, TFP	-0.03	-0.13	-0.20	-0.27	-0.16	0.02	0.42	0.36	0.28	0.20	0.05	-0.01	-0.07
Structures, TFP	-0.03	-0.11	-0.20	-0.23	-0.18	-0.09	0.13	0.16	0.19	0.21	0.24	0.08	-0.01
Equipment, GDP	-0.04	-0.11	-0.13	-0.10	0.06	0.24	0.58	0.50	0.25	0.15	-0.03	-0.13	-0.15
Structures, GDP	-0.09	-0.13	-0.15	-0.17	-0.03	0.06	0.34	0.32	0.33	0.27	0.22	0.10	-0.05
Panel B: HP-Filtered Data													
Equipment, TFP	-0.37	-0.38	-0.35	-0.24	-0.02	0.26	0.55	0.69	0.71	0.63	0.49	0.34	0.21
Structures, TFP	-0.36	-0.43	-0.47	-0.44	-0.34	-0.16	0.05	0.22	0.35	0.42	0.43	0.37	0.30
Equipment, GDP	-0.18	-0.11	0.01	0.18	0.40	0.63	0.80	0.80	0.66	0.47	0.24	0.03	-0.13
Structures, GDP	-0.35	-0.33	-0.26	-0.14	0.03	0.24	0.44	0.55	0.60	0.57	0.48	0.35	0.22
Panel C: Bandpass-Filtered Data													
Equipment, TFP	-0.48	-0.49	-0.44	-0.27	-0.01	0.30	0.58	0.77	0.81	0.72	0.55	0.37	0.21
Structures, TFP	-0.39	-0.47	-0.51	-0.47	-0.34	-0.15	0.08	0.30	0.45	0.52	0.50	0.44	0.35
Equipment, GDP	-0.26	-0.19	-0.06	0.14	0.38	0.61	0.76	0.78	0.66	0.44	0.19	-0.03	-0.20
Structures, GDP	-0.35	-0.35	-0.28	-0.16	0.02	0.22	0.40	0.53	0.57	0.53	0.42	0.27	0.13

Table 1.3: Length of Time (in Months) for Private Nonresidential Construction Projects, by Value and Type of Construction

This table reports the average number of months from start to completion for private nonresidential construction projects in 1990-91 and 2001-2015 by value and type of construction. Panels A and B show the length of time across value and type categories, respectively. Column 2001-15 shows the time-series average from 2001-02 to 2014-15. "Equal-weighted" measures the length of time as the simple average across projects without considering project costs, while "value-weighted" measures the cost-weighted length of time across projects. The data for 1990-91 are from Census Bureau (1992) and Montgomery (1995). Data for 2001-2015 are from Census Bureau (2016) (for equal-weighted numbers) and author calculation (for value-weighted numbers). See Appendix A.1 for more details.

Panel A: By Value of Construction

Values (thousands)	1990-91	Values (thousands)	2001-02	2014-15	2001-15
$10,000 or more	24.7	$10,000 or more	21.0	18.3	20.1
$5,000 - $9,999	17.4	$5,000 - $9,999	15.1	13.3	14.3
$3,000 - $4,999	14.4	$3,000 - $4,999	12.8	10.8	12.2
$1,000 - $2,999	11.4	$1,000 - $2,999	10.1	8.2	9.2
$250 - $999	7.2	$250 - $999	6.8	5.4	6.0
$75 - $249	4.1	$75 - $249	4.4	3.8	3.9
All (equal-weighted)	14.0	All (equal-weighted)	7.7	7.6	7.6
All (value-weighted)	**15.7**	All (value-weighted)	14.0	12.6	**13.6**

Panel B: By Type of Construction

Types	1990-91	Types	Equal-weighted			Value-weighted		
			2001-02	2014-15	2001-15	2001-02	2014-15	2001-15
Office	15.1	Office	6.9	6.9	6.8	13.5	11.6	12.7
Other Commercial	10.9	Commercial	6.5	6.1	6.3	10.8	9.3	10.0
Industrial	13.8	Manufacturing	9.0	11.3	9.2	17.5	16.1	15.8
Hospital & Institutional	19.1	Health care	9.6	9.6	9.4	16.1	14.6	15.8
Other	14.4	Educational	10.4	11.8	10.4	15.6	13.3	14.6
		Religious	11.0	10.3	11.1	16.0	14.5	15.2
		Amusement & recreation	9.7	8.3	9.2	14.9	12.5	14.1

Table 1.4: Return Predictability from Aggregate Investment Rates

This table reports in-sample and out-of-sample R^2 (in percent) for OLS predictions of US aggregate risk premium (from Kenneth French's website) from 1947Q1 to 2015Q4 across various horizons (H) ranging from 1 quarter to 20 quarters, $\sum_{h=1}^{H} R_{t+h} = a + b \ \mathrm{IK}_t + \varepsilon_{t+H}$. Predictor variables are US investment rates of nonresidential total (excluding intellectual property and products), nonresidential equipment, and nonresidential structures. The out-of-sample procedure uses the first half of the sample as the training period, then recursively tests and retrains in subsequent periods. b denotes the prediction slope coefficient. *p(NW)* denotes in-sample p-values constructed as in Newey and West (1987). Out-of-sample R^2 is calculated against historical averages of the predicted variable. *ENC-NEW* denotes the *New Encompassing* out-of-sample test statistic from Clark and McCracken (2001), following the construction methodology described in Kelly and Pruitt (2013). Significance for ENC-NEW statistics: $*** : p < 0.01$, $** : p < 0.05$, $* : p < 0.1$.

Investment Rates	H	In Sample			Out of Sample	
		$R^2\%$	b	$p(NW)$	$R^2\%$	ENC-NEW
Nonresidential	1	3.90	-6.48	0.002	0.68	3.242***
	4	11.24	-22.55	0.001	6.31	4.414***
	8	18.45	-39.39	0.000	15.52	5.079***
	12	29.02	-57.63	0.000	26.68	7.170***
	16	38.08	-73.34	0.000	33.98	9.340***
	20	39.26	-85.80	0.000	26.99	8.931***
Equipment	1	3.04	-2.93	0.005	-1.14	1.321*
	4	9.26	-10.45	0.003	1.10	2.196**
	8	15.52	-18.40	0.002	7.26	2.578**
	12	25.50	-27.38	0.000	18.48	4.351***
	16	35.16	-35.54	0.000	32.22	7.397***
	20	39.06	-42.57	0.000	34.73	9.520***
Structures	1	0.97	-3.19	0.091	-2.55	0.462
	4	2.42	-10.40	0.068	-6.47	0.099
	8	3.99	-18.43	0.051	-12.22	0.179
	12	6.44	-27.74	0.053	-26.44	0.326
	16	8.46	-35.97	0.087	-50.93	0.328
	20	7.83	-40.75	0.159	-87.25	0.063

Table 1.5: Return Predictability from Aggregate Investment Rates with Time-Varying Depreciation

This table reports in-sample and out-of-sample R^2 (in percent) for OLS predictions of US aggregate risk premium (from Kenneth French's website) from 1953Q1 to 2015Q4 across various horizons (H) ranging from 1 quarter to 20 quarters, $\sum_{h=1}^{H} R_{t+h} = a + b\ \mathrm{IK}_t + \varepsilon_{t+H}$. Predictor variables are US investment rates of nonresidential total (excluding intellectual property and products), nonresidential equipment, and nonresidential structures, constructed following Bachmann et al. (2013). See Appendix A.2 for details. The out-of-sample procedure uses the first half of the sample as the training period, then recursively tests and retrains in subsequent periods. b denotes the prediction slope coefficient. *p(NW)* denotes in-sample p-values constructed as in Newey and West (1987). Out-of-sample R^2 is calculated against historical averages of the predicted variable. *ENC-NEW* denotes the *New Encompassing* out-of-sample test statistic from Clark and McCracken (2001), following the construction methodology described in Kelly and Pruitt (2013). Significance for ENC-NEW statistics: $*** : p < 0.01,\ \ ** : p < 0.05,\ * : p < 0.1$.

Investment Rates	H	In Sample			Out of Sample	
		$R^2\%$	b	$p(NW)$	$R^2\%$	ENC-NEW
Nonresidential	1	2.26	-4.51	0.022	-2.24	1.323*
	4	6.45	-15.59	0.005	1.38	2.681**
	8	9.93	-26.12	0.003	8.68	2.544**
	12	17.15	-39.29	0.000	15.57	3.284***
	16	25.62	-52.25	0.000	22.53	4.163***
	20	27.74	-62.83	0.000	19.86	3.957***
Equipment	1	2.26	-2.78	0.023	-1.84	0.693
	4	6.15	-9.34	0.011	3.38	2.060**
	8	9.03	-15.17	0.013	8.67	1.679**
	12	17.14	-23.61	0.000	18.62	2.856**
	16	28.33	-32.49	0.000	34.72	5.598***
	20	32.71	-39.13	0.000	43.21	8.471***
Structures	1	1.10	-3.77	0.081	-1.40	0.856
	4	2.62	-11.95	0.072	-0.94	0.818
	8	4.04	-20.08	0.051	-1.03	0.507
	12	6.10	-28.39	0.076	-3.70	0.477
	16	7.46	-34.31	0.149	-10.97	0.343
	20	5.74	-34.85	0.272	-33.87	-0.248

Table 1.6: Return Predictability from Asset-Level Investment Rates

This table reports in-sample and out-of-sample R^2 (in percent) for OLS predictions of US aggregate risk premium (from Kenneth French's website) from 1947Q1 to 2015Q4 across various horizons (H) ranging from 4 quarters to 20 quarters, $\sum_{h=1}^{H} R_{t+h} = a + b \ \mathrm{IK}_t + \varepsilon_{t+H}$. Predictor variables are US investment rates of different types of nonresidential equipment and nonresidential structures. The out-of-sample procedure uses the first half of the sample as the training period, then recursively tests and retrains in subsequent periods. b denotes the prediction slope coefficient. *p(NW)* denotes in-sample p-values constructed as in Newey and West (1987). Out-of-sample R^2 is calculated against historical averages of the predicted variable. *ENC-NEW* denotes the *New Encompassing* out-of-sample test statistic from Clark and McCracken (2001), following the construction methodology described in Kelly and Pruitt (2013). Significance for ENC-NEW statistics: $***: p < 0.01$, $**: p < 0.05$, $*: p < 0.1$.

Investment Rates	H	In Sample			Out of Sample	
		R^2%	b	$p(NW)$	R^2%	ENC-NEW
Panel A: Equipment						
Information processing	4	6.41	-4.54	0.006	0.21	1.770**
	12	18.88	-12.69	0.001	0.98	2.128**
	20	28.25	-20.07	0.000	-36.63	1.576*
Industrial	4	6.51	-11.00	0.007	1.56	1.303*
	12	18.85	-29.58	0.000	10.55	2.066**
	20	34.05	-49.91	0.000	17.37	3.489***
Transportation	4	4.58	-5.02	0.015	-6.38	0.607
	12	13.91	-14.05	0.000	3.11	2.195**
	20	24.66	-23.49	0.000	19.76	5.199***
Other	4	8.22	-9.68	0.004	0.25	2.552**
	12	21.15	-24.57	0.000	20.25	4.751***
	20	35.60	-39.96	0.000	38.11	10.595***
Panel B: Structures						
Commercial and health care	4	2.91	-7.60	0.090	2.75	1.310*
	12	8.31	-20.70	0.074	0.12	0.503
	20	9.07	-28.47	0.166	-21.81	-0.037
Manufacturing	4	0.00	-0.30	0.947	-2.62	-0.474
	12	0.57	-5.73	0.558	-5.19	-0.298
	20	1.08	-9.56	0.544	-10.25	-0.136
Power and communication	4	9.55	-26.13	0.003	7.89	4.081
	12	12.01	-44.64	0.009	-5.02	1.332*
	20	6.69	-39.73	0.203	-13.02	1.089*
Mining exploration, shafts, and wells	4	0.02	0.31	0.908	-0.91	-0.170
	12	0.84	3.49	0.449	-6.07	-0.350
	20	1.58	5.73	0.330	-13.66	-0.451
Other	4	3.34	-19.01	0.038	4.48	1.390*
	12	8.98	-49.51	0.022	8.87	0.955
	20	12.64	-77.20	0.016	11.01	1.086*

Table 1.7: Return Predictability from Industry Investment Rates at 5-year Horizon

This table reports in-sample R^2 (in percent) for OLS predictions of US aggregate risk premium (Panel A) and of US 14 sectoral risk premium (Panel B) from 1962 to 2015 at a 5-year horizon, $\sum_{h=1}^{5} R_{t+h} = a + b\,\mathrm{IK}_t + \varepsilon_{t+5}$. Predictor variables are each industry's investment rates of equipment and structures. b denotes the prediction slope coefficient. *p(NW)* denotes in-sample p-values constructed as in Newey and West (1987). The last column shows the difference in R^2 between equipment and structures.

Industry	Equipment			Structures			ΔR^2
	$R^2\%$	b	$p(NW)$	$R^2\%$	b	$p(NW)$	E-S
Panel A: How Does Industry IK Predict **Aggregate** Risk Premium?							
Agriculture	7.25	-3.12	0.031	1.99	-6.03	0.184	5.27
Mining	0.12	-0.26	0.770	5.09	2.81	0.174	-4.97
Construction	14.32	-2.48	0.005	4.74	-1.58	0.259	9.57
Manufacturing	17.90	-7.01	0.003	11.96	-8.77	0.087	5.94
Wholesale	19.94	-3.19	0.001	0.26	-0.52	0.758	19.68
Retail	17.52	-5.07	0.000	9.30	-6.91	0.046	8.22
Transp & warehousing	20.95	-6.75	0.000	0.50	3.45	0.733	20.45
Information	18.23	-5.41	0.002	16.11	-11.21	0.029	2.12
Profes, scient & techn serv	6.04	-1.59	0.058	0.15	-0.38	0.829	5.89
Admin & waste manag serv	6.53	-2.41	0.100	0.02	0.18	0.921	6.51
Health care & social assist	6.57	-4.35	0.048	0.00	0.04	0.986	6.57
Arts, entert & recreation	5.94	-2.07	0.078	0.15	-0.58	0.814	5.79
Accomodation & food serv	7.92	-5.43	0.002	2.42	-2.17	0.214	5.51
Other serv, except govern	7.58	-2.47	0.169	0.63	1.51	0.597	6.95
Panel B: How Does Industry IK Predict **Industry** Risk Premium?							
Agriculture	0.76	-1.38	0.476	0.06	-1.34	0.796	0.71
Mining	9.67	-2.52	0.132	10.70	-4.88	0.049	-1.04
Construction	2.27	1.48	0.351	1.06	1.14	0.696	1.21
Manufacturing	13.60	-5.94	0.005	17.54	-8.92	0.021	-3.94
Wholesale	20.63	-3.41	0.019	6.13	-2.76	0.021	14.50
Retail	5.05	-3.24	0.184	14.72	-10.04	0.022	-9.66
Transp & warehousing	22.64	-6.33	0.001	1.04	4.60	0.639	21.60
Information	26.10	-7.70	0.000	8.10	-9.56	0.174	18.00
Profes, scient & techn serv	29.10	-4.49	0.000	3.36	-2.86	0.378	25.74
Admin & waste manag serv	12.63	-4.28	0.018	2.92	2.68	0.292	9.71
Health care & social assist	0.32	2.04	0.774	4.59	-5.09	0.444	-4.26
Arts, entert & recreation	1.23	1.75	0.407	0.47	2.05	0.714	0.77
Accomodation & food serv	5.58	-5.74	0.033	3.69	3.31	0.182	1.89
Other serv, except govern	1.55	-2.44	0.330	0.00	0.14	0.989	1.55

Table 1.8: Return Predictability from UK Aggregate Investment Rates

This table reports in-sample and out-of-sample R^2 (in percent) for OLS predictions of UK value-weighted market returns from 1970Q1 to 2013Q4 across various horizons (H) ranging from 1 quarter to 20 quarters, $\sum_{h=1}^{H} R_{t+h} = a + b \ \mathrm{IK}_t + \varepsilon_{t+H}$. Predictor variables are UK quarterly investment rates of nonresidential equipment and structures. See Appendix A.3 for more details on the data construction. The out-of-sample procedure uses the first half of the sample as the training period, then recursively tests and retrains in subsequent periods. b denotes the prediction slope coefficient. *p(NW)* denotes in-sample p-values constructed as in Newey and West (1987). Out-of-sample R^2 is calculated against historical averages of the predicted variable. *ENC-NEW* denotes the *New Encompassing* out-of-sample test statistic from Clark and McCracken (2001), following the construction methodology described in Kelly and Pruitt (2013). Significance for ENC-NEW statistics: $***: p < 0.01$, $**: p < 0.05$, $*: p < 0.1$.

Investment Rates	H	In Sample			Out of Sample	
		$R^2\%$	b	$p(NW)$	$R^2\%$	ENC-NEW
Nonresidential	1	0.67	-2.75	0.205	-0.06	0.182
	4	3.70	-13.54	0.024	5.41	1.596**
	8	5.44	-21.50	0.041	8.55	1.201*
	12	10.49	-35.42	0.003	20.62	2.881**
	16	17.16	-48.23	0.000	36.59	6.287***
	20	23.94	-55.84	0.000	41.71	8.089***
	24	28.60	-59.02	0.000	46.97	8.094***
Equipment	1	0.73	-1.90	0.178	-1.20	0.181
	4	2.50	-7.38	0.032	0.90	0.975
	8	2.38	-9.43	0.113	1.98	0.522
	12	4.36	-15.16	0.074	6.00	1.219*
	16	11.12	-25.99	0.020	21.33	3.535***
	20	19.71	-35.20	0.005	30.53	4.749***
	24	23.46	-37.25	0.003	33.18	3.513***
Structures	1	0.10	-1.23	0.682	-1.46	-0.443
	4	1.66	-10.35	0.166	2.94	0.551
	8	3.12	-18.56	0.091	2.88	0.345
	12	6.32	-31.07	0.020	8.71	0.766
	16	7.46	-35.57	0.001	13.46	1.109*
	20	8.66	-36.94	0.000	13.91	1.082*
	24	11.03	-40.16	0.000	19.09	1.301*

Table 1.9: Factor Predictability at 5-year Horizon from Aggregate Investment Rates

This table reports in-sample and out-of-sample R^2 (in percent) for OLS predictions of factor returns and portfolio risk premium at 5-year (20-quarter) horizon, $\sum_{h=1}^{20} R_{t+h} = a + b\ \mathrm{IK}_t + \varepsilon_{t+20}$. Predictor variables are US investment rates of nonresidential equipment and nonresidential structures. The forecasting targets include four factors from Fama and French (2016)'s 5-factor model (size, value, investment, and operating profitability) along with the corresponding decile portfolios (decile 1 and 10)—SMB (small-minus-big) with Size1 and Size10, HML (high-minus-low) with Value1 and Value10, CMA (conservative-minus-aggressive) with Inv1 and Inv10, RMW (robust-minus-weak) with OP1 and OP10; two factors from Hou et al. (2015)'s four-factor model—investment (IA) and profitability (ROE); and investment-minus-consumption portfolio (IMC) from Papanikolaou (2011). The sample for SMB, Size1, Size10, HML, Value1, and Value10 is 1947Q1-2015Q4. The sample for CMA, Inv1, Inv10, RMW, OP1 and OP10 is 1963Q3-2015Q4. The sample for IA and ROE is 1972Q1-2015Q4. And the sample for IMC is 1952Q1-2008Q4. The out-of-sample procedure uses the first half of the sample as the training period, then recursively tests and retrains in subsequent periods. b denotes the prediction slope coefficient. *p(NW)* denotes in-sample p-values constructed as in Newey and West (1987). Out-of-sample R^2 is calculated against historical averages of the predicted variable. *ENC-NEW* denotes the *New Encompassing* out-of-sample test statistic from Clark and McCracken (2001), following the construction methodology described in Kelly and Pruitt (2013). Significance for ENC-NEW statistics: $***: p < 0.01$, $**: p < 0.05$, $*: p < 0.1$.

Target	Equipment IK					Structures IK				
	In Sample			Out of Sample		In Sample			Out of Sample	
	R^2%	b	$p(NW)$	R^2%	ENC-NEW	R^2%	b	$p(NW)$	R^2%	ENC-NEW
SMB	7.87	16.11	0.007	4.14	0.956	0.96	12.06	0.519	-9.05	0.057
Size1	4.76	-21.62	0.147	-15.43	0.046	2.51	-33.59	0.375	-47.02	0.204
Size10	38.42	-46.00	0.000	30.17	5.946***	6.94	-41.82	0.185	-66.82	0.015
HML	3.02	7.01	0.256	1.74	0.181	15.03	33.42	0.013	16.91	2.253**
Value1	37.48	-50.91	0.000	37.44	9.543***	11.30	-59.78	0.040	-37.38	1.364*
Value10	12.92	-27.39	0.002	2.60	1.014*	0.00	0.62	0.987	-69.06	-1.056
CMA	25.00	18.91	0.011	22.96	3.678***	12.34	22.54	0.020	-0.27	1.167*
Inv1	8.42	-24.52	0.025	9.30	0.742	7.96	-40.46	0.200	-24.16	1.258*
Inv10	47.79	-73.15	0.000	59.30	13.794***	12.03	-62.29	0.019	-5.95	1.107*
RMW	0.99	3.63	0.602	-43.25	-0.989	2.90	-10.54	0.356	-3.24	0.281
OP1	27.86	-52.74	0.000	26.41	2.820**	1.24	-18.87	0.484	-12.23	-0.041
OP10	28.67	-40.77	0.000	37.62	4.063***	15.06	-50.15	0.076	-11.30	0.470
IA	19.09	13.92	0.003	15.03	1.487*	46.48	41.27	0.000	35.03	5.280***
ROE	0.01	-0.49	0.959	-14.61	-0.136	16.63	32.41	0.025	-0.46	0.027
IMC	24.13	-21.35	0.001	22.21	1.967**	11.73	-35.04	0.024	-2.79	1.292*

Table 1.10: Calibration

This table reports the calibrated values of parameters in the model. The model is calibrated at quarterly frequency.

Param	Name	Value
μ	GDP growth rate	0.0048
ρ_a	persistence of TFP	0.98
σ_a	volatility of TFP shock	0.01
β	time discount factor	0.995
γ	risk aversion	2
ρ_s	persistence of surplus consumption ratio	0.98
$\bar{S}$	steady state surplus consumption ratio	0.07
δ_e	depreciation rate of equipment	0.0338
δ_s	depreciation rate of structures	0.0077
α_e	production share of equipment	0.202
α_s	production share of structures	0.158
ν_e	equipment adjustment cost curvature	2
ν_s	structures adjustment cost curvature	2
η_e	equipment adjustment cost parameter	50
η_s	structures adjustment cost parameter	50
J_e	quarters of TTB for equipment	1
J_s	quarters of TTB for structures	5
ω_e	equipment project completion pattern	1
ω_s	structures project completion pattern	(0.10,0.15,0.20,0.25,0.30)

Table 1.11: Model Statistics for Macro Quantities and Asset Prices

This table reports the simulated model statistics for macro quantities and asset prices. Average statistics across 500 simulations are reported. Statistics for macro quantities are in quarterly values (volatility in percentage terms). Asset pricing statistics are in annualized percentage terms. Macro quantities are logged and first-differenced. $\sigma(\cdot)$, $\rho(\cdot)$, and $E(\cdot)$ denote volatility, correlation, and mean, respectively. The model scenarios include the Benchmark Model, Model 1 (same depreciation, $\bar{\delta}_e = \bar{\delta}_s = 0.25$), Model 2 (same production share, $\alpha_e = \alpha_s = 0.18$), Model 3 (no TTB, $J_e = J_s = 1$), Model 4 (No TTP, $\omega_i^s = 0.2, i = 1, ..., 5$), Model 5 (no habit, $H_t = 0$), Model 6 (no habit, no adjustment cost, $H_t = 0, \eta_e = \eta_s = 0$), Model 7 (no habit, no TTB, $H_t = 0, J_e = J_s = 1$), and Model 8 (no habit, no TTB, no adjustment cost, $H_t = 0, J_e = J_s = 1, \eta_e = \eta_s = 0$). Data statistics for macro quantities are calculated from NIPA quarterly sample from 1947Q1 to 2015Q4. Data statistics for asset prices are calculated from Kenneth French's data over the same sample deflated by CPI from BLS. $\Delta y, \Delta c, \Delta i, \Delta i_e$, and Δi_s are growth rates of output, consumption, investment, equipment investment, and structures investment, respectively. R_m is stock market return. R_f is risk-free rate. $R_m - R_f$ is risk premium.

Statistics	Data	Model Benchmark	Model 1 Same δ $\bar{\delta} = 0.025$	Model 2 Same α $\alpha = 0.18$	Model 3 No TTB $J = 1$	Model 4 No TTP $\omega_i^s = 0.2$	Model 5 No Habit	Model 6 No Habit No Adj	Model 7 No Habit No TTB	Model 8 No Habit No TTB, No Adj
Panel A: Macro Quantities										
$\sigma(\Delta y)$	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01
$\sigma(\Delta c)/\sigma(\Delta y)$	0.55	0.50	0.45	0.52	0.36	0.47	1.12	0.52	0.95	0.53
$\sigma(\Delta i)/\sigma(\Delta y)$	2.88	2.63	2.66	2.64	3.03	2.69	0.71	2.48	1.14	2.45
$\sigma(\Delta i_e)/\sigma(\Delta y)$	3.65	3.84	4.52	4.21	1.67	3.44	0.91	5.92	0.75	16.90
$\sigma(\Delta i_s)/\sigma(\Delta y)$	3.12	3.14	1.49	3.05	5.77	2.92	0.90	6.97	1.87	28.01
$\rho(\Delta y, \Delta c)$	0.48	0.97	0.98	0.97	0.99	0.98	1.00	1.00	1.00	1.00
$\rho(\Delta y, \Delta i)$	0.60	0.98	0.99	0.98	0.99	0.98	0.93	1.00	1.00	1.00
$\rho(\Delta y, \Delta i_e)$	0.58	0.93	0.97	0.92	0.99	0.94	1.00	0.78	1.00	0.71
$\rho(\Delta y, \Delta i_s)$	0.34	0.25	0.23	0.25	0.97	0.54	0.22	-0.21	1.00	-0.55
Panel B : Asset Prices										
$E(R_m)$	7.01	6.24	5.55	6.54	5.15	5.96	5.94	5.86	5.92	5.86
$\sigma(R_m)$	17.90	16.42	12.38	17.96	7.07	14.57	3.35	0.23	2.64	0.21
$E(R_f)$	0.57	1.92	2.56	1.71	3.81	2.21	5.77	5.88	5.80	5.86
$\sigma(R_f)$	2.52	5.84	2.74	7.02	0.54	4.78	0.48	0.21	0.20	0.20
$E(R_m - R_f)$	6.44	4.28	2.97	4.80	1.33	3.72	0.17	-0.02	0.12	0.00
$\sigma(R_m - R_f)$	17.66	15.01	11.85	16.15	6.91	13.51	3.26	0.08	2.59	0.07

Table 1.12: Return Predictability from Model-implied Investment Rates

This table reports model-implied in-sample R^2 (in percent) and regression slopes β for OLS predictions of aggregate risk premium, aggregate market return, and risk-free rate across various horizons (H) ranging from 1 quarter to 20 quarters, $\sum_{h=1}^{H} R_{t+h} = a + b\ \mathrm{IK}_t + \varepsilon_{t+H}$. Predictor variables are simulated equipment and structures investment rates. Note that model-implied investment rates are generated using simulated investment data and the perpetual inventory method, as the investment rates in the data are constructed. The model scenarios include Benchmark Model, Model 1 (same depreciation, $\bar{\delta}_e = \bar{\delta}_s = 0.025$), Model 2 (same production share, $\alpha_e = \alpha_s = 0.18$), and Model 3 (no TTB, $J_e = J_s = 1$).

Predictive Regressions	H	Data		Model Benchmark		Model 1 Same δ $\bar{\delta} = 0.025$		Model 2 Same α $\alpha = 0.18$		Model 3 No TTB $J = 1$	
		R^2%	b	R^2%	b	R^2%	b	R^2%	b	R^2%	b
Equipment	1	2.7	-2.8	7.2	-8.2	3.6	-4.3	8.6	-9.7	1.2	-1.7
Predicts	4	7.9	-9.8	20.9	-24.2	11.1	-13.4	24.6	-28.0	4.8	-6.6
R_m	12	21.0	-25.6	23.7	-30.0	17.9	-23.2	26.1	-32.2	13.1	-18.7
	20	33.1	-41.4	27.9	-37.2	24.1	-32.5	29.8	-38.6	20.0	-29.4
Equipment	1	3.0	-2.9	0.6	-1.8	0.7	-1.6	0.6	-1.8	0.7	-1.1
Predicts	4	9.3	-10.4	2.1	-6.9	2.6	-6.0	2.0	-6.8	2.7	-4.4
$R_m - R_f$	12	25.5	-27.4	5.5	-19.2	7.0	-16.5	5.0	-19.2	7.6	-12.1
	20	39.1	-42.6	8.8	-30.3	11.2	-25.8	8.0	-30.6	11.9	-18.7
Equipment	1	0.7	0.1	35.9	-6.4	30.7	-2.7	38.3	-7.9	35.4	-0.6
Predicts	4	2.0	0.7	32.9	-17.3	25.5	-7.5	35.9	-21.1	35.5	-2.3
R_f	12	2.5	1.8	5.9	-10.8	9.7	-6.7	6.0	-13.0	35.3	-6.6
	20	0.5	1.2	4.0	-6.9	9.9	-6.7	3.7	-8.1	34.9	-10.6
Structures	1	0.6	-2.4	1.1	-3.6	1.2	-2.6	1.1	-4.0	1.5	-1.7
Predicts	4	1.2	-7.3	2.6	-9.0	3.5	-7.9	2.2	-9.4	5.6	-6.7
R_m	12	3.0	-19.5	9.2	-19.8	11.0	-19.3	8.1	-20.0	15.5	-18.8
	20	3.7	-29.5	15.2	-29.1	17.2	-29.2	13.5	-29.1	23.6	-29.5
Structures	1	1.0	-3.2	0.8	-2.7	0.7	-1.8	0.9	-3.1	0.8	-1.2
Predicts	4	2.4	-10.4	3.2	-10.5	2.7	-6.9	3.3	-11.9	3.1	-4.5
$R_m - R_f$	12	6.4	-27.7	9.0	-29.3	7.7	-19.2	9.1	-33.3	8.8	-12.5
	20	7.8	-40.8	13.8	-44.8	11.9	-29.4	13.9	-51.0	13.8	-19.4
Structures	1	6.1	0.8	1.0	-0.8	4.0	-0.9	0.8	-0.9	40.2	-0.6
Predicts	4	10.6	3.1	1.1	1.5	3.4	-1.0	1.1	2.5	40.3	-2.2
R_f	12	13.1	8.2	5.1	9.4	7.0	-0.1	5.5	13.2	39.8	-6.3
	20	10.0	11.2	8.7	15.6	10.2	0.2	9.3	21.8	39.0	-10.1

Table 1.13: Return Predictability from Model-Implied Planned Investment

This table reports model-implied in-sample R^2 (in percent) and regression slopes β for OLS predictions of aggregate market return across various horizons ranging from 1 quarter to 20 quarters. Predictor variables are simulated log growth rates of the structures investment decision ($\log(X_{st}/X_{s,t-1})$), structures investment rate ($X_{st}/K_{s,t+4}$), and the ratio of structures investment decision to structures investment expenditures (X_{st}/I_{st}).

Predictor	Horizon	$R^2\%$	Slope	$p(NW)$
$\log(X_{st}/X_{s,t-1})$	1	0.95	-0.07	0.314
	4	9.80	-0.52	0.000
	8	6.06	-0.43	0.002
	12	5.37	-0.45	0.002
	16	5.00	-0.46	0.001
	20	4.60	-0.47	0.002
$X_{st}/K_{s,t+4}$	1	4.02	-6.26	0.014
	4	12.39	-18.95	0.003
	8	15.81	-22.83	0.008
	12	19.92	-27.97	0.011
	16	23.68	-32.84	0.015
	20	26.89	-37.14	0.017
X_{st}/I_{st}	1	6.79	-0.23	0.003
	4	25.13	-0.76	0.000
	8	18.35	-0.69	0.000
	12	16.54	-0.71	0.001
	16	15.45	-0.74	0.002
	20	14.54	-0.76	0.003

Table 1.14: VAR Analysis: Discount Rates versus Cash Flows

This table reports model-implied results for VAR analysis along with the empirical counterparts. Data are at annual frequency from 1947-2015. I use annual value-weighted CRSP market returns with and without dividends to back out the dividend-price ratio and then dividend growth (see Cochrane (2011) Appendix A). The model is simulated at quarterly frequency and aggregated to annual frequency. Median statistics from 500 simulations are reported. All variables, namely return (r), dividend growth (Δd), and dividend-price ratio (dp), are in logs. Panel A shows the regression slope coefficient, p value, and R^2 (in percent) for first-order VAR with dp_t as the right-hand variable. Panel B shows the long-run coefficients for long-run returns (r_t^{lr}) and dividend growth (Δd_t^{lr}) implied from the 1-year coefficients in Panel A. ρ is calculated as $\exp(-E(dp))/(1+\exp(-E(dp)))$. Panel C shows the variance components for dividend-price ratio both in raw value and in percentage of the variance in the dividend-price ratio ($\text{var}(dp_t)$). Due to the approximation error from Campell-Shiller decomposition, the sum of coefficients on r_t^{lr} and $-\Delta d_t^{lr}$ in Panel B approximately equals one, and the percentages of variance components sum to approximately 100%.

Panel A: First-Order VAR

	Data			Model		
Left-Hand Variable	Coeff	p	$R^2\%$	Coeff	p	$R^2\%$
r_{t+1}	0.11	0.018	7.05	0.12	0.003	11.14
Δd_{t+1}	0.02	0.608	0.55	-0.00	0.753	0.17
dp_{t+1}	0.94	0.000	90.89	0.90	0.000	80.88

Panel B: Long-Run Coefficients Implied by First-Order VAR

	Data	Model
Left-Hand Variable	Coeff	Coeff
$r_t^{lr} = \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}$	1.27	1.02
$\Delta d_t^{lr} = \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}$	0.27	0.01

Panel C: Variance Decomposition for Dividend-Price Ratio

	Data		Model	
	Value	Percent	Value	Percent
$\text{var}(dp_t)$	0.1506	100.00	0.1391	100.00
$\text{var}(r_t^{lr})$	0.2435	161.67	0.1307	104.46
$\text{var}(\Delta d_t^{lr})$	0.0110	7.31	0.0005	0.35
$-2\,\text{cov}(r_t^{lr}, \Delta d_t^{lr})$	-0.1035	-68.76	-0.0021	-2.58

Chapter 2

Asset Pricing and Risk Sharing with Limited Enforcement and Heterogeneous Preferences

2.1 Introduction

When a risk-sharing contract between agents cannot be fully enforced, the agent can be excluded from financial markets if he defaults on the contract (Kehoe and Levine (1993); Alvarez and Jermann (2000)). Alvarez and Jermann (2001) show that this contracting friction of limited enforcement generates limited risk sharing and a volatile pricing kernel, which matches asset pricing moments decently. The two preference parameters—risk aversion and time discount factor—play important roles in determining the amount of risk shared and the properties of the pricing kernel. A rise in the agent's risk aversion will increase the volatility of the pricing kernel directly, as risk aversion is the curvature parameter of marginal utility. But at the same time, more risk will be shared and individual consumption volatility decreases, resulting in a less volatile pricing kernel. Therefore, the overall effect is ambiguous. The effect of the time discount factor is similar. A rise in agent's time discount factor results in higher pricing kernel directly; however, it also decreases the pricing kernel through more risk sharing. When agents have different levels of risk aversion and time discount factors, how would risk sharing and the pricing kernel—and thus asset prices—change in contrast to when agents have the same preferences? Will they share more or less risk? Will the pricing kernel be lower or higher, less or more volatile? Will the equity premium be smaller or larger?

To answer these questions, I introduce heterogeneity in risk aversion and time discount factor into a two-agent endowment economy with enforcement constraints and aggregate and idiosyncratic income risk (Alvarez and Jermann (2001)), and study the corresponding implications for risk sharing and asset pricing.

First, I show that the relative time discount factor and the interaction between heterogeneous risk aversion and aggregate risk affect the evolution of the relative Pareto weight (RPW) of agents over time. When neither agent's enforcement constraint is binding, the RPW of the more patient agent goes up, and the RPW of the less risk-averse agent increases in booms and decreases in recessions. This is absent in Ligon et al. (2002), because in their economy there is no aggregate risk and agents have the same time preference. Ligon et al. prove that there exists an interval for the RPW to fall into for each state, and the RPW takes only boundary values of these intervals in the long run if risk sharing is limited. When heterogeneous preferences and aggregate risk are present, I show that the RPW takes not only boundary values of those intervals as in Ligon et al., but also certain values inside those intervals. In addition, the RPW does not go to zero or infinity—i.e., no agents die or dominate in the long run, since enforcement constraints entitle the agents to the option of autarky for all times, and agents can consume their non-zero endowment.

Next, I demonstrate that preference heterogeneity combined with limited enforcement generates a positive equity premium in a two-state example with only idiosyncratic income risk (i.e., no aggregate growth), while agents' endowment shares are symmetrically distributed. Enforcement constraints induce discount rate shocks as the marginal pricer changes over time, depending on which agent is not constrained. When agents have symmetric endowment and the same preference parameters, the pricing kernel or stochastic discount factor (SDF) is also symmetric across agents. Although the marginal pricing agent is changing, the conditional SDF does not vary. Thus there is no time variation in price-dividend ratios, and the equity premium is zero. Introducing heterogeneous preferences breaks down the symmetry of the SDF across agents. The conditional SDF varies, depending on which agent is the marginal pricer, which results in time-varying price-dividend ratios and positive equity premium. In contrast to homogeneous risk aversion, the amount of risk sharing increases little for the low-risk-aversion agent, but decreases dramatically for the high-risk-aversion agent, leading to high consumption volatility for the latter. When the more risk-averse agent is unconstrained and becomes the marginal pricer, the SDF is large and volatile, resulting in a sizable equity premium. As for the case of heterogeneous time preference, the more patient agent has a greater chance to be the marginal pricer due to his higher patience level, even though the two agents have symmetric endowment distributions. As a result, the SDF is more affected by the more patient agent. When he cannot trade away most of his income risk with the less patient agent, equity premium is high.

Last, I use the recursive Lagrangian method of Marcet and Marimon (2016) to solve a calibrated model with both idiosyncratic and aggregate risk. As the intuition of the two-state example carries through, I show that heterogeneous preferences generate asymmetric risk sharing across agents and lead to a higher and more volatile equity premium than homogeneous preferences when agents are subjected to enforcement constraints. Undesirably, the risk-free rate could be too volatile, because the discount rate shocks induced by enforcement constraints become asymmetric with preference heterogeneity and lead to more variable SDF[1]. In addition, heterogeneous time preference shows more promise than heterogeneous risk aversion for better matching asset pricing moments. In particular, the former could generate 7.05% mean equity premium and 27.87% equity volatility, with moderate risk aversion around 3 and reasonable heterogeneous time discount factors 0.85 and 0.75 for the two agents.

The paper's contribution to asset pricing literature is to show that idiosyncratic income

[1] I use CRRA (constant relative risk aversion) utility. Heterogeneous risk aversion implies heterogeneous EIS (elasticity of intertemporal substitution). The problem might be solved by Epstein-Zin preference, which separates EIS and relative risk aversion so that agents can have heterogeneous risk aversion, but the same EIS at the same time. I do not pursue it here, however, because of the associated computational difficulties for the combination of Epstein-Zin and enforcement constraints.

risk and incomplete markets become more relevant for asset prices when agents have heterogeneous preferences[2]. Alvarez and Jermann (2001) and Krueger and Lustig (2010) show that idiosyncratic income risk does not affect (multiplicative) equity premium if its distribution is independent of aggregate risk and aggregate risk is i.i.d. over time. I show that preference heterogeneity renders discount rate shocks asymmetric across agents induced by enforcement constraints, leading to time variation in conditional SDF and price-dividend ratios, and thus non-zero equity premium even without any aggregate risk. Quantitatively, heterogeneous time preference is better than homogeneous preference at matching asset pricing moments when aggregate risk is present[3]. As for the paper's contribution to risk sharing literature, I generalize the theoretical result of Ligon et al. (2002) on the evolution of the RPW over time under limited enforcement to include preference heterogeneity and aggregate risk. In particular, I show that the relative time discount factor and the interaction between heterogeneous risk aversion and aggregate uncertainty affect the evolution of the RPW.

The paper is related to several strands of literature. First, it is related to the literature on heterogeneous preferences and asset prices. For early contributions, see Dumas (1989) and Wang (1996). The two papers, along with Basak and Cuoco (1998), draw an undesirable implication: with positive growth, less risk-averse agents will dominate the economy in the long run. To ensure stationarity, the literature has introduced habit into preferences (Chan and Kogan (2002), Xiouros and Zapatero (2010), and Bhamra and Uppal (2014)) or overlapping generations with agents vanishing each period (Gomes and Michaelides (2008) and Gârleanu and Panageas (2015))[4,5]. In this paper, enforcement constraints naturally emerge as a device to ensure stationary long-run distribution, as agents always have the option to choose autarky and would not end up with zero consumption. In addition, the papers cited

[2]See Cochrane (2017) for an overview of macro asset pricing literature. For widely used asset pricing models, see Campbell and Cochrane's (1999) external habit model, Bansal and Yaron's (2004) long run risk model, and Barro's (2006) rare disaster model.

[3]Albuquerque et al. (2015) show that a representative agent model with exogenous time preference shocks accounts for key asset pricing moments. I interpret time preference shocks in my model as endogenously driven by enforcement constraints, as the marginal pricing agent who is not constrained varies over time.

[4]For other contributions, see Guvenen (2009), Chabakauri (2015), Coen-Pirani (2004), Kogan et al. (2007), Longstaff and Wang (2012), and Lengwiler (2005).

[5]There is also a literature on heterogeneous beliefs and asset prices. Borovika (2016) shows that when agents have heterogeneous beliefs, Duffie-Epstein-Zin preferences lead to long-run outcomes in which both agents survive or more incorrect agents dominate. See the paper's references for heterogeneous beliefs.

above examine how heterogeneous preferences affect asset prices with only aggregate risk[6]. In contrast, I include both aggregate risk and idiosyncratic risk and show explicitly how the two risks and preference heterogeneity are intertwined through the evolution of the RPW to determine consumption allocations, risk sharing, and asset prices.

Second, the paper relates to the contracting friction of limited enforcement. For early contributions, see Kehoe and Levine (1993), Kocherlakota (1996), and Alvarez and Jermann (2000). Alvarez and Jermann (2001) study its implications for asset pricing. Krueger et al. (2008) test the SDF generated from limited enforcement and find support in U.S. consumption data. Beker and Espino (2015) incorporate heterogeneous beliefs into Alvarez and Jermann (2001) to explain return momentum and reversals. I also build on Alvarez and Jermann (2001) and examine how preference heterogeneity affects risk sharing and asset prices. Chien and Lustig (2010) show that less risk sharing can be sustained by allowing agents to file for bankruptcy instead of excluding them from financial markets forever, and this improves the asset pricing predictions. Ai and Bhandari (2016) study the asset pricing implications of uninsurable tail risk in labor productivities when markets are endogenously incomplete due to principal-side limited commitment. Cao (2014) shows that agents with incorrect beliefs survive by holding on to their nonfinancial wealth under limited commitment. For risk-sharing implications, Ligon et al. (2002) find some support for limited enforcement using Indian village consumption data. Laczó (2015) introduces preference heterogeneity into Ligon et al. (2002) and finds evidence for heterogeneous preferences using Indian village consumption data and structural estimation[7]. Krueger and Perri (2006) study the implications of limited enforcement for consumption inequality. Rampini and Viswanathan (2016) show that limited enforcement can explain a household's insurance pattern. In addition, see Kehoe and Perri (2002) for an international application.

The paper is also related to the literature on how incomplete markets and portfolio constraints affect asset prices. Mankiw (1986) and Constantinides and Duffie (1996) show that equity premium will increase if the cross-sectional volatility of non-tradable idiosyncratic risk is higher in recessions[8]. The main difference is that in my model the markets

[6] Gomes and Michaelides (2008) is an exception.

[7] There is no aggregate risk in Laczó's model. And her focus is risk sharing; I study asset prices as well.

[8] For other early contributions, see He and Modest (1995), Luttmer (1996), Telmer (1993), Heaton and

are endogenous incomplete due to endogenous solvency constraints, while the literature usually assumes exogenous incomplete markets due to limited securities to trade, trade frictions, or exogenous borrowing constraints, etc. Storesletten et al. (2007) extend the endowment economy of Constantinides and Duffie (1996) to overlapping generations, along with capital accumulation. They show that idiosyncratic risk inhibits the intergenerational sharing of aggregate risk, but capital accumulation mitigates it by providing self-insurance. Chabakauri (2013) finds that tighter margins and leverage constraints generate higher risk premia. Rytchkov (2014) finds that state-dependent and time-varying margin constraints reduce risk-free rate, but increase risk premium.

The paper proceeds as follows. Section 2 outlines the model. Section 3 uses a two-state example to gain the intuition of the model. Section 4 presents calibration and quantitative results, and Section 5 concludes.

2.2 Model

In this section, I outline the model. I first adopt the promised utility approach to formulate the contracting problem, derive the evolution of the RPW, and show its long-run properties. I then use the recursive Lagrangian method of Marcet and Marimon (2016) (see also Kehoe and Perri (2002)) to set up the planner's problem and present a computation algorithm to solve the consumption allocations. Last, I decentralize the economy, following Alvarez and Jermann (2000), and pin down the asset prices with the solved allocations.

2.2.1 Environment

Two agents are endowed with a random stream of income e_{it}, $i = 1, 2$. Aggregate endowment is $e_t = e_{1t} + e_{2t}$ and grows over time $g_{t+1} = e_{t+1}/e_t$. Agents' income share is $\hat{e}_{it} = e_{it}/e_t$. I assume that $z = (g, \hat{e})$ jointly follows a finite-state Markov process. I denote $z^t = (z_0, z_1, z_2, ..., z_t)$ as the history up to t. Transition probabilities from $t-1$ to t are denoted as $\pi(z^t|z^{t-1})$.

Agents have CRRA utility, but may differ in the relative risk aversion coefficient and

Lucas (1996), and Zhang (1997).

time discount factor. The utility function is

$$u^i(c_{it}) = \frac{c_{it}^{1-\gamma_i}}{1-\gamma_i},$$

and lifetime utility is defined as

$$U_t^i(c_i) \equiv E_t \sum_{j=0}^{\infty} \beta_i^j u^i(c_{it+j}),$$

where c_{it} is agent i's consumption at time t, γ_i is the relative risk aversion coefficient, and β_i is the time discount factor. Contracts are not fully enforceable. If the agent defaults, he will be excluded from financial markets forever and remain in autarky. Agents' consumption choices satisfy the participation constraints,

$$U_t^i(c_i) \geq U_t^i(e_i), \quad i = 1, 2, \quad t = 0, 1, 2, ...,$$

where $U_t^i(e_i)$ is the utility value of autarky starting from time t. Note that for notational convenience, I write out the states z^t only when it is necessary to avoid confusion.

2.2.2 Promised Utility Formulation

I take the promised utility approach to formulate the contracting problem. Pareto frontiers should satisfy

$$V(w,z) = \max_{c_i, w'(z')} \{u(c_1) + \beta_1 \sum_{z' \in Z} \pi(z'|z) V(w'(z'), z')\}$$

$$\begin{aligned}
& c_1 + c_2 \leq e, \\
\lambda: \quad & u(c_2) + \beta_2 \sum_{z' \in Z} \pi(z'|z) w'(z') \geq w, \\
\beta_2 \pi(z'|z) \eta^2(z'): \quad & w'(z') \geq U^2(z'), \qquad z' \in Z, \\
\beta_1 \pi(z'|z) \eta^1(z'): \quad & V(w'(z'), z') \geq U^1(z'), \qquad z' \in Z,
\end{aligned}$$

where V is the lifetime utility of agent 1, w is the promised life-time utility to agent 2, λ is the Lagrangian multiplier on the promise-keeping constraint, and $\eta^i(z')$ is the state-dependent

Lagrangian multiplier on agent i's participation constraints.

The first-order conditions are

$$\begin{aligned} \frac{u_c^1(c_1)}{u_c^2(c_2)} &= \lambda, \\ -V'(w'(z'), z') &= \frac{\lambda + \eta^2(z')}{1 + \eta^1(z')} \frac{\beta_2}{\beta_1}, \end{aligned} \tag{2.1}$$

together with the envelope condition

$$-V'(w, z) = \lambda.$$

There is aggregate growth in the economy. To obtain a stationary economy, I normalize some variables as follows:

$$\begin{aligned} \hat{c}_{it} &\equiv c_{it}/e_t, \quad \hat{e}_{it} \equiv e_{it}/e_t, \\ \hat{w} &\equiv \frac{w}{e^{1-\gamma_2}}, \quad \hat{V}(\hat{w}, z) \equiv \frac{V(w, z)}{e^{1-\gamma_1}}, \\ \hat{U}^i(z) &\equiv \frac{U^i(z)}{e^{1-\gamma_i}}, \\ \hat{\pi}_i(z'|z) &\equiv \frac{\pi(z'|z) g(z')^{1-\gamma_i}}{\sum_{z'} \pi(z'|z) g(z')^{1-\gamma_i}}, \\ \hat{\beta}_i(z) &\equiv \beta_i \sum_{z'} \pi(z'|z) g(z')^{1-\gamma_i}. \end{aligned}$$

The original recursive problem can then be rewritten as

$$\hat{V}(\hat{w}, z) = \max_{\hat{c}_i, \hat{w}'(z')} \{u(\hat{c}_1) + \hat{\beta}_1 \sum_{z' \in Z} \hat{\pi}_1(z'|z) \hat{V}(\hat{w}'(z'), z')\}$$

$$\begin{aligned} & \hat{c}_1 + \hat{c}_2 \leq 1, \\ \hat{\lambda}: \quad & u(\hat{c}_2) + \hat{\beta}_2 \sum_{z' \in Z} \hat{\pi}_2(z'|z) \hat{w}'(z') \geq \hat{w}, \\ \hat{\beta}_2 \hat{\pi}_2(z'|z) \hat{\eta}^2(z'): \quad & \hat{w}'(z') \geq \hat{U}^2(z'), \qquad z' \in Z, \\ \hat{\beta}_1 \hat{\pi}_1(z'|z) \hat{\eta}^1(z'): \quad & \hat{V}(\hat{w}'(z'), z') \geq \hat{U}^1(z'), \qquad z' \in Z, \end{aligned}$$

where $\hat{\lambda}$ and $\hat{\eta}^i(z')$ are Lagrangian multipliers on the constraints after normalization.

The first-order conditions are

$$\frac{u_c^1(\hat{c}_1)}{u_c^2(\hat{c}_2)} = \hat{\lambda}, \tag{2.2}$$

$$\begin{aligned} -\hat{V}'(\hat{w}'(z'), z') &= \frac{\hat{\lambda} + \hat{\eta}^2(z')}{1 + \hat{\eta}^1(z')} \frac{\hat{\beta}_2 \hat{\pi}_2}{\hat{\beta}_1 \hat{\pi}_1} \\ &= \frac{\hat{\lambda} + \hat{\eta}^2(z')}{1 + \hat{\eta}^1(z')} \frac{\beta_2}{\beta_1} g(z')^{\gamma_1 - \gamma_2}, \end{aligned} \tag{2.3}$$

together with the envelope condition

$$-\hat{V}'(\hat{w}, z) = \hat{\lambda}. \tag{2.4}$$

It is easy to see from equations 2.1 and 2.2 that

$$\hat{\lambda} = \lambda / e_t^{\gamma_2 - \gamma_1}.$$

$\hat{\lambda}$ equals the marginal utility of consumption *share*, which is also the (normalized or preference-adjusted) RPW (of agent 2 with respect to agent 1) in the planner's problem, which will be discussed in subsection 2.2.4[9]. Combining equations 2.3 and 2.4, I obtain

$$\hat{\lambda}' = \frac{\hat{\lambda} + \hat{\eta}^2(z')}{1 + \hat{\eta}^1(z')} \frac{\beta_2}{\beta_1} g(z')^{\gamma_1 - \gamma_2}. \tag{2.5}$$

The above equation describes how idiosyncratic risk, aggregate risk, and preference heterogeneity are intertwined to influence RPW and, thus, consumption allocations, risk sharing, and asset prices. When agent i gets a high idiosyncratic income shock tomorrow, his participation constraint binds, $\hat{\eta}^i > 0$, and his RPW rises. When both agents are not constrained, $\hat{\eta}^1 = \hat{\eta}^2 = 0$, the RPW of the more patient agent will increase due to the term β_2/β_1, and the RPW of the less risk-averse agent will rise in booms and decline in recessions due to the term $g(z')^{\gamma_1 - \gamma_2}$.

[9] For convenience, I call $\hat{\lambda}$ RPW hereafter.

2.2.3 Risk Sharing

I characterize the evolution of $\hat{\lambda}$ over time and its long-run properties.

Proposition 2. *Suppose agents can have heterogeneous preferences. A constrained-efficient contract can be characterized as follows: There exist S state-dependent intervals $[\underline{\lambda}_s, \bar{\lambda}_s]$, $s = 1, 2, ..., S$ such that given $\hat{\lambda}_t$ and next period occurring state s, $\hat{\lambda}_{t+1}$ updates as:*

$$\hat{\lambda}_{t+1} = \begin{cases} \underline{\lambda}_s, & \textit{if} \quad \hat{\lambda}_t g_s^{\gamma_1-\gamma_2} \frac{\beta_2}{\beta_1} < \underline{\lambda}_s \\ \hat{\lambda}_t g_s^{\gamma_1-\gamma_2} \frac{\beta_2}{\beta_1}, & \textit{if} \quad \hat{\lambda}_t g_s^{\gamma_1-\gamma_2} \frac{\beta_2}{\beta_1} \in [\underline{\lambda}_s, \bar{\lambda}_s] \\ \bar{\lambda}_s, & \textit{if} \quad \hat{\lambda}_t g_s^{\gamma_1-\gamma_2} \frac{\beta_2}{\beta_1} > \bar{\lambda}_s. \end{cases}$$

Proof. The proof follows Ligon et al. (2002). First, there exist S state-dependent intervals $[\underline{V}_s, \bar{V}_s]$ and $[\underline{w}_s, \bar{w}_s]$ for agent 1's and agent 2's possible lifetime utility values, following Thomas and Worrall (1988). Obviously, $\underline{V}_s = \hat{U}_s^1$ and $\underline{w}_s = \hat{U}_s^2$. Second, let $\underline{\lambda}_s = -\hat{V}'(\underline{w}_s)$ and $\bar{\lambda}_s = -\hat{V}'(\bar{w}_s)$. Then considering the three cases

- If $\hat{\lambda}_t g_s^{\gamma_1-\gamma_2} \frac{\beta_2}{\beta_1} < \underline{\lambda}_s$, $\hat{\lambda}_{t+1} \in [\underline{\lambda}_s, \bar{\lambda}_s] > \hat{\lambda}_t g_s^{\gamma_1-\gamma_2} \frac{\beta_2}{\beta_1}$. Equation 2.5 then implies $\hat{\eta}_s^2 > 0$. Thus $\hat{w}_s = \underline{w}_s$. Hence $\hat{\lambda}_{t+1} = -\hat{V}'(\underline{w}_s) = \underline{\lambda}_s$.

- If $\hat{\lambda}_t g_s^{\gamma_1-\gamma_2} \frac{\beta_2}{\beta_1} > \bar{\lambda}_s$, $\hat{\lambda}_{t+1} \in [\underline{\lambda}_s, \bar{\lambda}_s] < \hat{\lambda}_t g_s^{\gamma_1-\gamma_2} \frac{\beta_2}{\beta_1}$. Equation 2.5 then implies $\hat{\eta}_s^1 > 0$. Thus $\hat{V}_s = \underline{V}_s$ and $\hat{w}_s = \bar{w}_s$. Hence $\hat{\lambda}_{t+1} = -\hat{V}'(\bar{w}_s) = \bar{\lambda}_s$.

- If $\hat{\lambda}_t g_s^{\gamma_1-\gamma_2} \frac{\beta_2}{\beta_1} \in [\underline{\lambda}_s, \bar{\lambda}_s]$, it must have $\hat{\eta}_s^1 = \hat{\eta}_s^2 = 0$, and hence $\hat{\lambda}_{t+1} = \hat{\lambda}_t g_s^{\gamma_1-\gamma_2} \frac{\beta_2}{\beta_1}$. Suppose the contrary, $\hat{\eta}_s^2 > 0$ and $\hat{\eta}_s^1 = 0$. Then $\hat{w}_s = \underline{w}_s$ and $\hat{\lambda}_{t+1} = \underline{\lambda}_s$. But equation (2.5) implies $\hat{\lambda}_{t+1} > \hat{\lambda}_t g_s^{\gamma_1-\gamma_2} \frac{\beta_2}{\beta_1}$. Contradiction. The symmetric argument holds for $\hat{\eta}_s^2 = 0$ and $\hat{\eta}_s^1 > 0$. And it is not possible that $\hat{\eta}_s^2 > 0$ and $\hat{\eta}_s^1 > 0$.

□

Corollary 1. *Suppose agents have the same preferences, i.e., $\beta_1 = \beta_2, \gamma_1 = \gamma_2$. Given $\hat{\lambda}_t$*

and next period occurring state s, $\hat{\lambda}_{t+1}$ *updates as:*

$$\hat{\lambda}_{t+1} = \begin{cases} \underline{\lambda}_s, & \textit{if } \hat{\lambda}_t < \underline{\lambda}_s \\ \hat{\lambda}_t, & \textit{if } \hat{\lambda}_t \in [\underline{\lambda}_s, \bar{\lambda}_s] \\ \bar{\lambda}_s, & \textit{if } \hat{\lambda}_t > \bar{\lambda}_s. \end{cases}$$

This is the same result as in Ligon et al. (2002). When there are no enforcement constraints, the first best is achieved and $\hat{\lambda}$ is constant over time, $\hat{\lambda}_{t+1} = \hat{\lambda}_t$ for all t. With participation constraints present, next period $\hat{\lambda}$ remains unchanged if possible, and changes the minimum amount to be inside the interval of the possible values of (preference-adjusted) RPW if this period $\hat{\lambda}$ is outside the interval. When heterogeneous preferences are present, the extra term $g_s^{\gamma_1-\gamma_2}\frac{\beta_2}{\beta_1}$ shows up in the updating rule of $\hat{\lambda}$.

Proposition 3. *Assume agents have heterogeneous preferences.*

(i) Both agents survive in the long run, i.e., $\lim_{T\to\infty}\hat{\lambda}_T \neq 0$ *and* $\lim_{T\to\infty}\hat{\lambda}_T \neq \infty$.

(ii) There exists no constrained efficient contract that features constant consumption shares.

(ii) When the constrained efficient contract features limited risk sharing, the long-run ergodic set of the RPW is several certain boundary $\hat{\lambda}$*s plus sets of other points inside* $\hat{\lambda}$ *intervals.*

Proof. (i) From Proposition 2, $\lim_{T\to\infty}\hat{\lambda}_T \in [\min_s \underline{\lambda}_s, \max_s \bar{\lambda}_s]$. Thus, $\lim_{T\to\infty}\hat{\lambda}_T \neq 0$ and $\lim_{T\to\infty}\hat{\lambda}_T \neq \infty$.

(ii) From Corollary 1, if there is an overlapping interval for all $[\underline{\lambda}_s, \bar{\lambda}_s]_{s=1}^S$, a constrained efficient contract achieves constant consumption under homogeneous preferences. From Proposition 2, even if an overlapping interval exists, there is some positive probability that $\lim_{T\to\infty}\hat{\lambda}_T$ will be outside the interval under heterogeneous preferences, due to the term $g_s^{\gamma_1-\gamma_2}\frac{\beta_2}{\beta_1}$. Hence there exists no constrained efficient contract that features constant consumption shares.

(ii) From Corollary 1, $\lim_{T\to\infty}\hat{\lambda}_T$ will only take values from $\{\underline{\lambda}_s, \bar{\lambda}_s\}_{s=1}^S$ when agents have homogeneous preferences. From Proposition 2, $\lim_{T\to\infty}\hat{\lambda}_T$ will take values not only from $\{\underline{\lambda}_s, \bar{\lambda}_s\}_{s=1}^S$ but also inside $[\underline{\lambda}_s, \bar{\lambda}_s]_{s=1}^S$ due to the term $g_s^{\gamma_1-\gamma_2}\frac{\beta_2}{\beta_1}$, when agents have heterogeneous preferences. $\square$

2.2.4 Recursive Lagrangian Formulation

One difficulty inherent to computation associated with the promised utility approach is finding the maximum promised utility $\bar{w}_s$ for each state s, which is endogenous determined. Thus I resort to the relatively manageable recursive Lagrangian approach—introducing the RPW as the co-state variable—to compute the model solutions. The section on the promised utility approach is retained to facilitate proofs for the evolution of the RPW.

Given initial Pareto weight ϕ_i, the planner's problem is

$$\max_{\{c_{1t},c_{2t}\}} \left[E_0 \sum_{t=0}^{\infty} \sum_{i=1,2} \beta_i^t \phi_i u^i(c_{it}) \right]$$

$$U_t^i(c_i) \geq U_t^i(e_i), \quad i = 1, 2, \quad t = 0, 1, 2, ...$$

$$c_{1t} + c_{2t} = e_t, \quad t = 0, 1, 2, ...$$

I follow Marcet and Marimon (2016) (see also Kehoe and Perri (2002)) and use a recursive Lagrangian formulation to solve the problem. Let the Lagrange multiplier on the enforcement constraints be $\{\mu_{it}\}_{t=0}^{\infty}$. The Lagrangian is

$$\mathcal{L} = \sum_{i=1,2} E_0 \sum_{t=0}^{\infty} \beta_i^t \left[\phi_i u^i(c_{it}) + \mu_{it} \left[E_t \sum_{j=0}^{\infty} \beta_i^j u^i(c_{it+j}) - U_t^i(e_i) \right] \right].$$

By Abel's summation, $\mathcal{L}$ can be simplified as

$$\mathcal{L} = \sum_{i=1,2} E_0 \sum_{t=0}^{\infty} \beta_i^t \left[H_{it} u^i(c_{it}) - \mu_{it} U_t^i(e_i) \right]$$

$$H_{it} = H_{it-1} + \mu_{it}$$

$$H_{i,-1} = \phi_i,$$

where H_{it} is the time-varying Pareto weight for agent i at t and it equals the sum of initial Pareto weight ϕ_i and all historical multipliers up to t, $\{\mu_{is}\}_{s=0}^{t}$. First-order conditions wrt. consumption imply

$$\frac{u_c^1(c_{1t})}{u_c^2(c_{2t})} = \frac{H_{2t}}{H_{1t}} \left(\frac{\beta_2}{\beta_1} \right)^t,$$

where $u_c^i(c_{it})$ denotes the marginal utility of consumption for agent i. Redefine the multipliers,

$$\lambda_t \equiv \frac{H_{2t}}{H_{1t}} \left(\frac{\beta_2}{\beta_1}\right)^t, \quad \nu_{it} \equiv \frac{\mu_{it}}{H_{it}},$$

where λ_t is the RPW of agent 2 to agent 1. The law of motion for this can then be derived as

$$\lambda_t = \frac{1-\nu_{1t}}{1-\nu_{2t}} \lambda_{t-1} \frac{\beta_2}{\beta_1}.$$

When agent i's enforcement constraint binds, $\nu_{it} > 0$, his RPW goes up. When both agents are not constrained, $\nu_{it} = 0$ $i = 1, 2$, the more patient (higher β) agent has an increase in his RPW.

Definition 1. *Given initial Pareto weights ϕ_i, constrained efficient allocations for the growth economy are $\{c_1, c_2, \nu_1, \nu_2, \lambda\}$, which satisfy equations* (2.6), (2.7), *complementary slackness condition* (2.8), *and resource constraint* (2.9)*:*

$$\frac{u_c^1(c_{1t})}{u_c^2(c_{2t})} = \lambda_t, \tag{2.6}$$

$$\lambda_t = \frac{1-\nu_{1t}}{1-\nu_{2t}} \lambda_{t-1} \frac{\beta_2}{\beta_1}, \tag{2.7}$$

$$\lambda_{-1} = \frac{\phi_2}{\phi_1},$$

$$\nu_{it} \geq 0, \qquad \nu_{it}\left[U_t^i(c_i) - U_t^i(e_i)\right] = 0, \tag{2.8}$$

$$c_{1t} + c_{2t} = e_t. \tag{2.9}$$

To obtain a stationary economy, I normalize some variables as follows:

$$\hat{c}_{it} \equiv c_{it}/e_t, \quad \hat{e}_{it} \equiv e_{it}/e_t,$$

$$\hat{\lambda}_t \equiv \lambda_t / e_t^{\gamma_2 - \gamma_1}, \quad \hat{\lambda}_{-1} \equiv \lambda_{-1}, (e_{-1} \equiv 1)$$

$$\hat{U}_t^i(\hat{c}_i) \equiv \frac{U_t^i(c_i)}{e_t^{1-\gamma_i}}, \quad \hat{U}_t^i(\hat{e}_i) \equiv \frac{U_t^i(e_i)}{e_t^{1-\gamma_i}},$$

$$\hat{\pi}_i(z_{t+1}|z_t) \equiv \frac{\pi(z_{t+1}|z_t) g_{t+1}^{1-\gamma_i}}{\sum_{z_{t+1}} \pi(z_{t+1}|z_t) g_{t+1}^{1-\gamma_i}},$$

$$\hat{\beta}_i(z_t) \equiv \beta_i \sum_{z_{t+1}} \pi(z_{t+1}|z_t) g_{t+1}^{1-\gamma_i}.$$

Definition 2. *Given the initial Pareto weights ϕ_i, constrained efficient allocations for the stationary economy are $\{\hat{c}_1, \hat{c}_2, \nu_1, \nu_2, \hat{\lambda}\}$, which satisfy equations* (2.10), (2.11), *complementary slackness condition* (2.12), *and resource constraint* (2.13)*:*

$$\frac{u_c^1(\hat{c}_{1t})}{u_c^2(\hat{c}_{2t})} = \hat{\lambda}_t, \tag{2.10}$$

$$\hat{\lambda}_t = \frac{1-\nu_{1t}}{1-\nu_{2t}} \hat{\lambda}_{t-1} g_t^{\gamma_1-\gamma_2} \frac{\beta_2}{\beta_1}, \tag{2.11}$$

$$\hat{\lambda}_{-1} = \frac{\phi_2}{\phi_1},$$

$$\nu_{it} \geq 0, \qquad \nu_{it}\left[\hat{U}_t^i(\hat{c}_i) - \hat{U}_t^i(\hat{e}_i)\right] = 0, \tag{2.12}$$

$$\hat{c}_{1t} + \hat{c}_{2t} = 1. \tag{2.13}$$

It is easy to derive the equivalence between the growth economy and stationary economy using the redefined variables. The recursive Lagrangian formulation is consistent with the promised utility formulation. In particular, $\hat{\lambda}$, the normalized RPW in (2.11), is exactly the marginal utility ratio of consumption *share* in (2.5). One difference is that $\hat{\lambda}$ is implied by the envelope condition in the promised utility formulation, while it is a state variable in the recursive Lagrangian formulation.

2.2.5 Computation

Denote state variables $x = \{\hat{\lambda}, z\}$, where z is the joint Markov process of idiosyncratic uncertainty and aggregate uncertainty, and the added co-state variable $\hat{\lambda}$ is the RPW of agent 2 wrt. agent 1. Denote the set of policy functions and value functions as $F(x) = \{\hat{c}_i(x), \hat{\lambda}'(x), \nu_i(x), W_i(x)\}, i = 1, 2$, where

$$W_i(x) = u(\hat{c}_i(x)) + \beta_i(z) \sum_{z'} \pi_i(z'|z) W_i(x').$$

The computation algorithm follows several steps:

1. Set up a grid X over the state space.

2. Set the full risk-sharing solution as the initial guess $F^0(x)$.

3. For each $x \in X$, guess that neither enforcement constraint binds.

   3a. If satisfied, set the new policies and value functions $F^1(x)$ to be $F^0(x)$

   3b. If agent 1's or agent 2's constraint is not satisfied, impose the binding constraint and recalculate the solution as $F^1(x)$.

4. Iterate until $|F^n(x) - F^{n-1}(x)| < \epsilon$.

Linear interpolation is used for states not on the grid points. See Appendix B.1 for more details.

2.2.6 Decentralization and Asset Prices

I follow Alvarez and Jermann (2000) to decentralize the economy[10].

Definition 3. ***A competitive equilibrium with solvency constraints*** *$\{B_i\}$ that is not too tight for initial conditions $\{a_{i,0}\}$ has quantities $\{a_i\}$ and prices $\{q\}$ s.t.*

1. For each i, given $\{a_{i,0}\}$ and prices $\{q\}$, $\{c_i, a_i'\}$ solves

$$\begin{aligned} J_i\left(a_i(z^t)\right) &= \max\left\{u^i\left(c_i(z^t)\right) + \beta_i E\left[J_i\left(a_i(z^{t+1})\right)\right]\right\} \\ c_i(z^t) + \sum_{z^{t+1}|z^t} a_i(z^{t+1})q(z^{t+1}|z^t) &= e_i(z^t) + a_i(z^t), \\ a_i(z^{t+1}) &\geq B_i(z^{t+1}), \end{aligned} \tag{2.14}$$

where a_i is agent i's Arrow security holdings, q is the price of Arrow security, B_i is agent i's endogenous determined borrowing constraint, and J_i is agent i's value function.

2. Good market and asset markets clear,

$$\sum_i c_i(z^t) = e(z^t)$$

$$\sum_i a_i(z^{t+1}) = 0$$

[10] Preference heterogeneity does not affect proofs for the welfare theorems.

3. Solvency constraint is not-too-tight,

$$J_i\left(B_i(z^{t+1})\right) = U^i_{t+1}(e_i).$$

The Euler equation can be derived as

$$q_t(z^{t+1}|z^t) = \beta_i \pi(z^{t+1}|z^t)\frac{u^i_c(c_{it+1}(z^{t+1}))}{u^i_c(c_{it}(z^t))} + \frac{\zeta_{it+1}(z^{t+1})}{u^i_c(c_{it}(z^t))},$$

where $\zeta_{it+1}(z^{t+1})$ is the Lagrangian multiplier on the solvency constraint (2.14). Since the two agents cannot have binding constraints at the same time, it follows that $\zeta_{1t+1} = 0$ or $\zeta_{2t+1} = 0$ or both. Thus

$$q_t(z^{t+1}|z^t) = \pi(z^{t+1}|z^t)\max_{i=1,2}\left(\beta_i \frac{u^i_c(c_{it+1}(z^{t+1}))}{u^i_c(c_{it}(z^t))}\right). \tag{2.15}$$

The agent whose solvency constraint is not binding has the highest intertemporal marginal rate of substitution (IMRS) and prices the Arrow security at that state.

The asset pricing equation is

$$E_t[M_{t+1}R_{t+1}] = 1,$$

where the SDF M_{t+1} is

$$M_{t+1} = \max_{i=1,2}\left(\beta_i \frac{u^i_c(c_{i,t+1})}{u^i_c(c_{i,t})}\right) = \max_{i=1,2}\left(\beta_i \left(\frac{\hat{c}_{i,t+1}}{\hat{c}_{i,t}}\right)^{-\gamma_i}\left(\frac{e_{t+1}}{e_t}\right)^{-\gamma_i}\right). \tag{2.16}$$

The risk-free rate is

$$R_{f,t+1} = 1/E_t[M_{t+1}]$$

and the return of aggregate consumption claim is

$$R_{s,t+1} = \frac{P_{t+1} + e_{t+1}}{P_t} = \frac{1 + \frac{P_{t+1}}{e_{t+1}}}{\frac{P_t}{e_t}}\frac{e_{t+1}}{e_t}$$

where
$$\frac{P_t}{e_t} = E_t\left[M_{t+1}\left(1 + \frac{P_{t+1}}{e_{t+1}}\right)\frac{e_{t+1}}{e_t}\right].$$

P_t is the price of aggregate consumption claim. And the equity premium is defined as

$$R_{e,t+1} = R_{s,t+1} - R_{f,t+1}.$$

I use the solution of consumption allocations from the planner's problem to pin down the SDF, then the price-consumption ratio and asset returns.

2.3 Two-state Example

I introduce preference heterogeneity into the two-state example from Alvarez and Jermann (2001)[11]. I show that it induces discount rate shocks, which—combined with the limited risk sharing generated by enforcement constraints—generate a positive and volatile equity premium with the absence of aggregate risk[12].

Figure 2.1 compares the long-run consumption allocation under homogeneous risk aversion with that under heterogeneous risk aversion, where agent 2 has a higher risk aversion (2.7) than agent 1 (1.5). Agents have the same time discount factor, which is fixed at $\beta = 0.65$. With the same risk aversion, the two agents have symmetric consumption shares because transition probability and endowment shares are symmetric. The upper solid line shows the consumption share when the agent has a higher income share, while the lower dashed line shows the consumption share for a lower income share. As risk aversion increases, agents change from autarky to limited risk sharing to full risk sharing. The squares (for agent 1) and diamonds (for agent 2) present a particular case in which agents have different risk aversion parameters, i.e., $\gamma_1 = 1.5$ and $\gamma_2 = 2.7$. The more risk-averse agent 2 pays a premium to the less risk-averse agent 1 for insurance so that agent 2 consumes less than agent 1 in both states ($c_{1L} > c_{2L}$ and $c_{1H} > c_{2H}$). In addition, agent 1's consumption allocation deviates little from autarky if they have had the same low risk aversion of 1.5, but much less risk is shared for agent 2 than when they have the same high risk aversion of

[11] The transition probability and endowment shares are

$$\Pi = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}, \begin{bmatrix} \hat{e}_1(s_1) \\ \hat{e}_1(s_2) \end{bmatrix} = \begin{bmatrix} 0.641 \\ 0.359 \end{bmatrix}, \begin{bmatrix} \hat{e}_2(s_1) \\ \hat{e}_2(s_2) \end{bmatrix} = \begin{bmatrix} 0.359 \\ 0.641 \end{bmatrix}.$$

[12] Without aggregate risk, the equity premium is essentially the term premium of a perpetual consol bond.

2.7.

[Insert Figure 2.1 about here]

[Insert Figure 2.2 about here]

Figure 2.2 compares the long-run consumption allocation under homogeneous time discount factor with that under heterogeneous time discount factor where agent 1 has a higher patience level (0.7) than agent 1 (0.5). Agents have the same time risk aversion coefficient, which is fixed at $\gamma = 3.0$. Similarly, the two agents have symmetric consumption shares when they have the same patience level β. The upper solid (lower dashed) line shows the consumption share when the agent has a higher (lower) income share. As β rises, agents change from autarky to limited risk sharing to full risk sharing. The squares (for agent 2) and diamonds (for agent 1) present a particular heterogeneous β case $\beta_1 = 0.7$ and $\beta_2 = 0.5$. The more patient agent 1 pays a premium to the less patient agent 2 for insurance, so that agent 1 consumes less than agent 2 in both states ($c_{1L} < c_{2L}$ and $c_{1H} < c_{2H}$).

When agents have the same preference, there will be no risk premium because agents are symmetric and there is no aggregate risk. Although the agent who prices the asset is time-varying, the conditional SDF, and thus price-(aggregate) consumption ratio, do not change due to the symmetry assumption. But when agents have heterogeneous preferences, a positive risk premium instead results. Preference heterogeneity breaks the symmetry of SDF and induces conditional variation in SDF, and thus in price-consumption ratio. Therefore, preference heterogeneity generates asymmetric discount rate shocks and leads to positive equity premium, even without aggregate risk.

[Insert Table 2.1 about here]

Table 2.1 dissects how heterogeneous preference produces a positive equity premium. Panel A contrasts the case of heterogeneous γ with homogeneous γ while keeping β fixed at 0.65. When agents have the same γ (1.5 or 2.7), there is no variation in conditional SDF due to the symmetric transition probability matrix and symmetric endowment shares. Thus the stock return equals the risk-free rate ($R_s = R_f$) state by state, and there is no equity premium. When agents have different γ, however, the symmetry in SDF breaks down. Especially for the transition from $s_t = s_2$ to $s_{t+1} = s_1$, which is priced by the more risk-averse agent 2, SDF is much higher because he has high consumption volatility and higher risk aversion. This boosts the mean and volatility of SDF. Despite no aggregate risk, conditional variation in SDF generates variation in the price-consumption ratio, resulting in a positive equity premium. Panel B contrasts the case of heterogeneous β with homogeneous β while keeping γ fixed at 3.0. The results are similar to those in Panel A. When agents have the same β (0.7 or 0.5), there is no variation in conditional SDF. Thus $R_s = R_f$ state by state, and there is no equity premium. But when agents have different β, the symmetry in SDF breaks down. Especially for the transition from $s_t = s_1$ to $s_{t+1} = s_2$, which is priced by the more patient agent 1, SDF is much higher due to high consumption volatility and higher patience. In addition, agent 1 is also the marginal pricer for the transition from $s_t = s_2$ to $s_{t+1} = s_2$, as he has a larger IMRS (larger β and no consumption change for either agent). Although aggregate risk is not present, conditional variation in SDF generates variation in the price-consumption ratio and positive equity premium as a result.

2.4 Quantitative Results

2.4.1 Calibration

I assume that there are four states and agents are symmetric in their endowment processes. I follow the high β annual calibration of Alvarez and Jermann (2001) with 10 moments to pin down 10 parameters, including 2 aggregate growth rates, 2 idiosyncratic income shares, 2 aggregate transition probabilities, and 4 idiosyncratic transition probabilities. See Appendix B.2 for details. For preference parameters β and γ, I pick values around the numbers used in Alvarez and Jermann (2001) ($\beta = 0.78$ and $\gamma = 3.5$). I simulate the model 1,000 times and 5,000 periods each time, with the first 500 periods burned out to obtain the long-run distribution.

2.4.2 Heterogeneous Risk Aversion

Figure 2.3 shows how the degree of risk sharing measured by consumption volatility and SDF change when agents have heterogeneous risk aversion γ, but the same time preference

β. I compare the case in which both agents have the same risk aversion of 2.5 or 4 with the case in which one agent has risk aversion $\gamma_1 = 2.5$ and the other has $\gamma_2 = 4$ for a range of β. When β is small, risk sharing incentive is low. Agents stay in autarky and consumption volatility is high. As β increases, the degree of risk sharing rises. When β becomes large enough, agents achieve full risk sharing. The degree of risk sharing for the heterogeneous risk aversion case lies between the case of both low risk aversion and the case of both high risk aversion for most β. In particular, the amount of risk sharing does not increase much for the less risk-averse agent 1, but decreases a fair amount for the more risk-averse agent 2, compared with the case of homogeneous risk aversion. In addition, agent 2's consumption profile is more volatile than agent 1's. Note that when $\beta > 0.96$, heterogeneous risk aversion generates less risk sharing for both agents than homogeneous risk aversion. Even when β is large enough, constant consumption shares (i.e., zero standard deviation for consumption shares) cannot be achieved, as proved by Proposition 3.

[Insert Figure 2.3 about here]

The magnitude of β affects SDF through two opposing channels. On the one hand, higher β leads to higher and more volatile SDF directly given individual consumption growth. On the other hand, higher β induces more risk sharing, and thus lower consumption volatility, reducing the size and volatility of SDF. When β is small, agents stay in autarky and only the first effect is present. Therefore, the mean and volatility of SDF increase with β. As β becomes larger, the second effect dominates and the mean and volatility of SDF decline with β. For a range of moderate β, heterogeneous risk aversion generates higher and more volatile SDF than homogeneous risk aversion. Specifically, when β is between 0.78 and 0.85, heterogeneous agents share limited risk, and the more risk-averse agent with volatile consumption pushes up the mean and volatility of SDF in the states in which he is not constrained and is the marginal pricer.

[Insert Figure 2.4 about here]

Figure 2.4 shows how the mean and volatility of asset returns change when agents have heterogeneous risk aversion but the same time preference. The two effects of β on SDF are present inversely on the risk-free rate. The mean of R_f ($E(R_f)$) first declines and then increases with β. But the volatility of R_f ($\sigma(R_f)$) declines all the way down. This is because the direct effect of β on $\sigma(R_f)$ always dominates the indirect effect. When β lies in between 0.78 and 0.85, limited risk is shared and heterogeneous risk aversion produces lower $E(R_f)$ than homogeneous risk aversion. For most ranges of β, heterogeneity in risk aversion generates higher equity premium and higher equity volatility, because the more risk-averse agent 2 feels unsafe holding stocks in recessions when his income volatility is higher and he cannot trade away most of his income risk with the less risk-averse agent 1. In addition, $\sigma(R_f)$ is much higher when agents have different risk aversion. This results in an overly volatile risk-free rate.

2.4.3 Heterogeneous Time Preference

Figure 2.5 shows how the degree of risk sharing and SDF change when agents have heterogeneous time preference but the same risk aversion. I compare the case in which both agents have the same time preference β of 0.75 or 0.85 with the case in which agent 1 has β of 0.85, while agent 2 has 0.75 for each γ. When γ is small, risk sharing incentive is low. Agents stay in autarky and consumption volatility is high. As γ increases, the degree of risk sharing rises. When γ becomes large enough, agents achieve full risk sharing. The degree of risk sharing for the heterogeneous β case lies between the case of both low β and the case of both high β for $\gamma \in [3, 4.2]$. Specifically, the amount of risk sharing does not increase much for the less patient agent 2, but decreases a fair amount for the more patient agent 1, in contrast to the corresponding case of homogeneous β. In addition, agent 2's consumption profile is more volatile than agent 1's. Note that when $\gamma > 4.7$, the heterogeneous time discount factor generates less risk sharing for both agents than homogeneous time discount factors do. Even when γ is large enough, constant consumption shares cannot be achieved, as proved by Proposition 3.

[Insert Figure 2.5 about here]

The magnitude of γ affects SDF through two opposing channels. On the one hand, higher γ leads to higher and more volatile SDF directly given individual consumption growth. On the other hand, higher γ induces more risk sharing and thus smaller individual consumption growth, reducing the size and volatility of SDF. When γ is small, agents stay in autarky and only the first effect is present. Therefore, the mean and vol of SDF increase with γ. As γ becomes larger, the second effect dominates and the mean and volatility of SDF decline with γ. For most γ, heterogeneous β generates a higher mean of SDF, because the more patient agent will price more states including all transitions with no state change $s_{t+1} = s_t$, and thus no consumption change. Yet the volatility of SDF for heterogeneous β is smaller than for same low β when $\gamma \in [2.9,\ 4.7]$, because agents have higher consumption volatility in the latter case.

[Insert Figure 2.6 about here]

Despite the low SDF volatility, heterogeneous β produces a high and volatile equity premium, as shown in Figure 2.6. There is a spike for mean equity premium $E(R_e)$ and equity return volatility $\sigma(R_s)$ at γ values where agents change from autarky to little risk sharing. This is because when the more patient agent 1 receives a higher income share (and thus is not constrained) and prices the assets, he requires a high compensation for bearing risk, as little of his income risk can be traded away with the less patient agent 2. The pattern of the mean risk-free rate $E(R_f)$ corresponds inversely with the mean of SDF $E(M)$. Heterogeneous β generates variation in conditional SDF, leading to an excessively volatile risk-free rate when γ is large. As demonstrated by the two-state example, preference heterogeneity in γ or β renders discount rate shocks asymmetric, generates conditional SDF variation, and results in volatile asset returns.

2.4.4 Asset Pricing Moments

Table 2.2 presents the simulation moments for asset prices and consumption allocation. Panels A and B show the effect of heterogeneous risk aversion and heterogeneous time preference, respectively. For Panel A, β is chosen to match the mean of risk-free rate $E(R_f)$ as closely as possible after γ is chosen around the value 3.5 from Alvarez and Jermann (2001). For Panel B, γ is chosen to match $E(R_f)$ as closely as possible after β is chosen around the value 0.78 from Alvarez and Jermann (2001). Panel A shows that heterogeneity in γ ($\gamma_1 = 2.5, \gamma_2 = 4.0$) increases the mean equity premium and equity volatility, 4.11% and 12.70%, respectively, in comparison with homogeneous low risk aversion ($\gamma_1 = \gamma_2 = 2.5$), 1.59% and 6.64%. But heterogeneous risk aversion does not necessarily improve over homogeneous high risk aversion ($\gamma_1 = \gamma_2 = 4.0$), where equity premium is slightly higher (4.52%) and equity volatility is slightly lower (10.74%) in the latter. In addition, the risk-free rate is too volatile (10.93%). Panel B shows that heterogeneity in β ($\beta_1 = 0.85, \beta_2 = 0.75$) does not necessarily increase SDF volatility, but produces a much higher equity premium (7.05%) and more volatile equity returns (27.87%) than homogeneous β (3.99% and 9.94% for $\beta_1 = \beta_2 = 0.75$ and 2.09% and 7.42% for $\beta_1 = \beta_2 = 0.85$). Moreover, risk-free rate volatility rises little (6.96% vs. 6.14% and 4.08%), although it is already more volatile than in the data (4.01%). Alvarez and Jermann (2001) matches the volatility of risk-free rate 5.67% from Mehra and Prescott (1985). Given that I follow their calibration, the heterogeneous β case shows much promise for better matching the mean and volatility of the equity premium. That the risk-free rate is too volatile is because the main mechanism of the model originates from the discount rate shocks induced by enforcement constraints, which become asymmetric with the presence of preference heterogeneity. I mention a potential remedy for this in footnote 2.1.

[Insert Table 2.2 about here]

[Insert Table 2.3 about here]

As proved by Alvarez and Jermann (2001) and Krueger and Lustig (2010), when the distribution of idiosyncratic shocks is independent of aggregate shocks and aggregate shocks are i.i.d. (in short, as independent risk), the consumption share $\hat{c}$ does not depend on aggregate uncertainty. Hence the term premium is zero and the multiplicative equity premium is the same as in a representative agent economy. Table 2.3 shows that heterogeneous preferences generate a positive term premium and higher equity premium than homogeneous preferences when risk is independent[13]. Panel A shows the results for heterogeneous risk aversion when β is fixed at 0.5. When agents have the same $\gamma = 2.5$ or 3.5, the term

[13]The risk aversion and time discount factor parameters are chosen for qualitative illustration, not to match moments quantitatively.

premium is zero and the multiplicative equity premium is small (less than 0.5%). But when agents have different risk aversion ($\gamma_1 = 2.5$ and $\gamma_2 = 3.5$), the term premium becomes 0.72% and the multiplicative equity premium rises to 2.44%. But all returns become quite volatile. Panel B shows the results for the case of heterogeneous time discount factor when γ is fixed at 3.0. Similarly, when agents have the same $\beta = 0.45$ or 0.55, the term premium is zero and the equity premium is small (less than 0.5%). But when agents have heterogeneous time preference ($\beta_1 = 0.55$ and $\beta_2 = 0.45$), a small difference in $\beta(0.1)$ causes a 4.37% term premium and a 7.72% multiplicative equity premium. In addition, the returns of equity and perpetual bond are much more volatile than the risk-free rate. In contrast to the case of heterogeneous γ, heterogeneous β can generate a higher and more volatile equity premium without inducing too much volatility in the risk-free rate.

[Insert Table 2.4 about here]

In the benchmark model, all agents' income is labor income; agents are not endowed with any assets at the beginning. I relax this assumption by letting the agents be endowed with both labor income and half of a Lucas tree initially. The Lucas tree bears fruits as dividend income, which is a constant fraction of total income, $\omega = D_t/e_t$[14]. The rest $1 - \omega$ fraction of e_t is labor income, and the agents' shares of labor income are subject to idiosyncratic shocks, as in the benchmark model. In default, the Lucas tree will be seized and the agents will consume only labor income in autarky. Table 2.4 shows how asset prices change as ω varies from zero to 10%[15]. Asset prices are very sensitive to the magnitude of ω. As it increases, autarky becomes less attractive and agents share more risk. The volatility of consumption share and SDF decrease. The risk-free rate becomes much higher, while the equity premium declines a lot, from 3.45% to 0.63%, 4.11% to 0.72%, and 7.05% to 2.59%, for homogeneous preferences, heterogeneous risk aversion, and heterogeneous time discount factor, respectively. In order for limited enforcement to matter more for the equity premium, I could alleviate the punishment for default, such as allowing agents to trade a risk-free bond as in Krueger and Perri (2006), or allowing agents to come back to financial

[14] Other parameter values are not changed. The only difference, therefore, is that $\omega = 0$ in the benchmark model while it is positive here.

[15] Chien et al. (2012) calibrate the fraction of collateralizable income ω to be 10%.

markets several years after defaulting. These less severe punishment mechanisms, instead of permanent exclusion from financial markets, can reduce optimal risk sharing and increase the equity premium[16].

2.5 Conclusion

I introduce heterogeneous preferences (heterogeneity in risk aversion and time discount factor) to a two-agent endowment economy with enforcement constraints and aggregate and idiosyncratic uncertainty (Alvarez and Jermann (2001)), and study the corresponding asset pricing and risk sharing implications. I find that the relative time discount factor and the interaction between heterogeneous risk aversion and aggregate uncertainty affect the evolution of the relative Pareto weight of agents. I demonstrate that preference heterogeneity can generate a positive equity premium with only idiosyncratic uncertainty present, since the conditional pricing kernel is time-varying depending on which agent is the marginal pricer. With the calibrated model, I show that preference heterogeneity boosts the mean and volatility of the equity premium quantitatively, when the more risk-averse or the more patient agent cannot trade away most of his income risk with the other agent. In particular, heterogeneous time preference holds great promise for a better matching of key asset pricing moments.

[16] Another way to boost the equity premium is to view equity as a levered aggregate consumption claim.

Figure 2.1: Consumption Share in the Two-state Example: Homogeneous γ vs. Heterogeneous γ

This figure shows the long-run stationary consumption shares of agents 1 and 2 in the two-state example. The horizontal axis denotes risk aversion and the vertical axis consumption shares. The agents' time discount factors are the same, and fixed at $\beta = 0.65$. The upper solid (lower dashed) line depicts how the agent's consumption share changes with risk aversion when he receives a high (low) endowment realization under homogeneous risk aversion ($\gamma_1 = \gamma_2$). The squares and diamonds present a particular case in which agents have different risk aversion coefficients, i.e., $\gamma_1 = 1.5$ and $\gamma_2 = 2.7$. Squares denote the consumption share of the less risk-averse agent 1 ($\gamma_1 = 1.5$) when he receives a high (upper square) or low (lower square) endowment realization. Diamonds are for the more risk-averse agent 2 ($\gamma_2 = 2.7$). $c_{1L} = 0.377$, $c_{1H} = 0.628$, $c_{2L} = 0.372$, $c_{2H} = 0.623$.

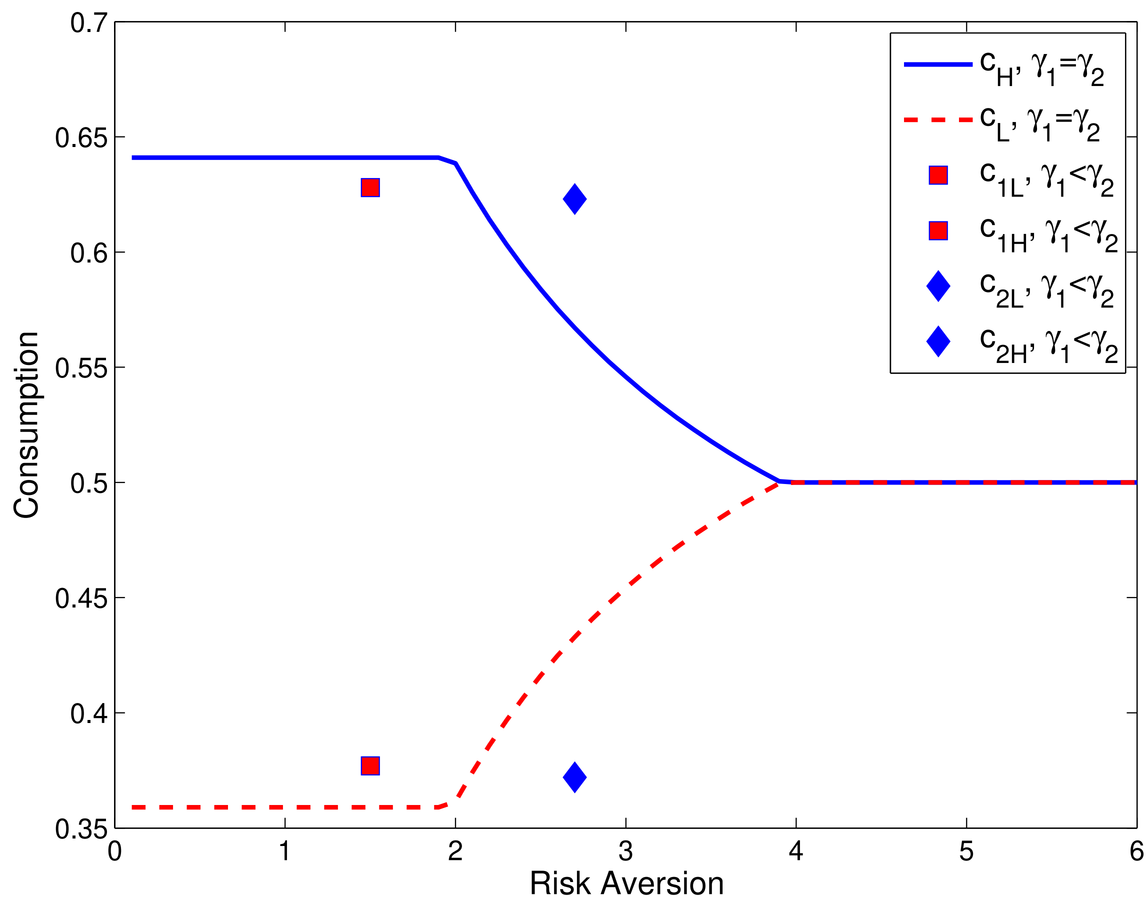


Figure 2.2: Consumption Share in the Two-state Example: Homogeneous β vs. Heterogeneous β

This figure shows the long-run stationary consumption shares of agents 1 and 2 in the two-state example. The horizontal axis denotes time discount factor and the vertical axis consumption shares. The agents' risk aversion coefficients are the same, and fixed at $\gamma = 3.0$. The upper solid (lower dashed) line depicts how the agent's consumption share changes with the time discount factor when he receives a high (low) endowment realization under homogeneous time discount factor ($\beta_1 = \beta_2$). The squares and diamonds present a particular case in which agents have different time discount factors, i.e., $\beta_1 = 0.7$ and $\beta_2 = 0.5$. Squares denote the consumption share of the less patient agent 2 ($\beta_2 = 0.5$) when he receives a high (upper square) or low (lower square) endowment realization. Diamonds are for the more patient agent 1 ($\beta_1 = 0.7$). $c_{1L} = 0.422,\ c_{1H} = 0.531,\ c_{2L} = 0.469,\ c_{2H} = 0.578.$

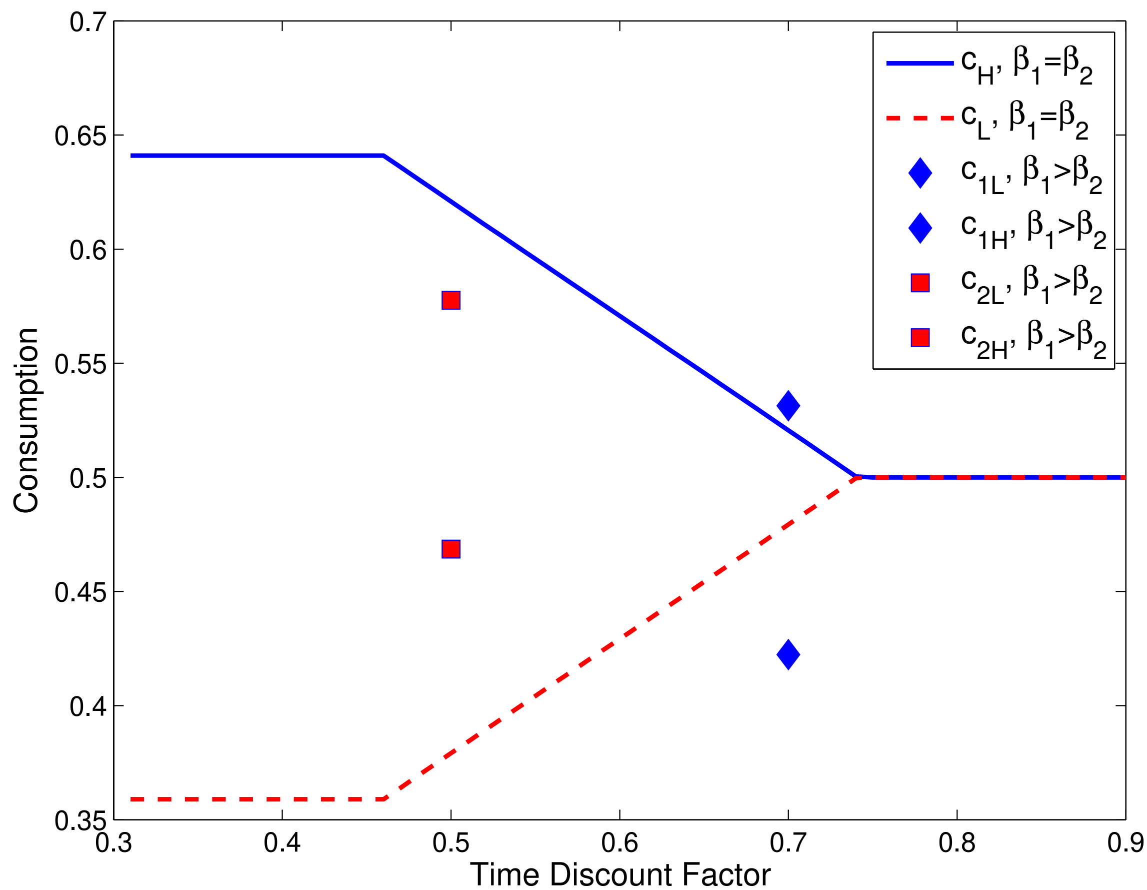


Figure 2.3: Risk Sharing: Heterogeneous vs. Homogeneous Risk Aversion

This figure shows how agents' consumption shares and the resulting SDF change with time discount factor β. Agents have the same β, but may have different risk aversion coefficients γ. The upper part of the figure presents the standard deviation of consumption shares, which measures the degree of risk sharing. The diamond-marked line indicates that both agents have the same risk aversion $\gamma = 2.5$, while the square-marked line the same $\gamma = 4.0$. The other two lines present the case of heterogeneous risk aversion, $\gamma_1 = 2.5$ (the dashed line for agent 1) and $\gamma_2 = 4.0$ (the solid line for agent 2). The more risk-averse agent 2 has less volatile consumption than the less risk-averse agent 1 when there is non-zero risk sharing. The lower part of the figure presents the mean and standard deviation of SDF. The diamond-marked (square-marked) line stands for homogeneous risk aversion $\gamma = 2.5$ ($\gamma = 4.0$), while the solid line stands for heterogeneous risk aversion $\gamma_1 = 2.5$ and $\gamma_2 = 4.0$.

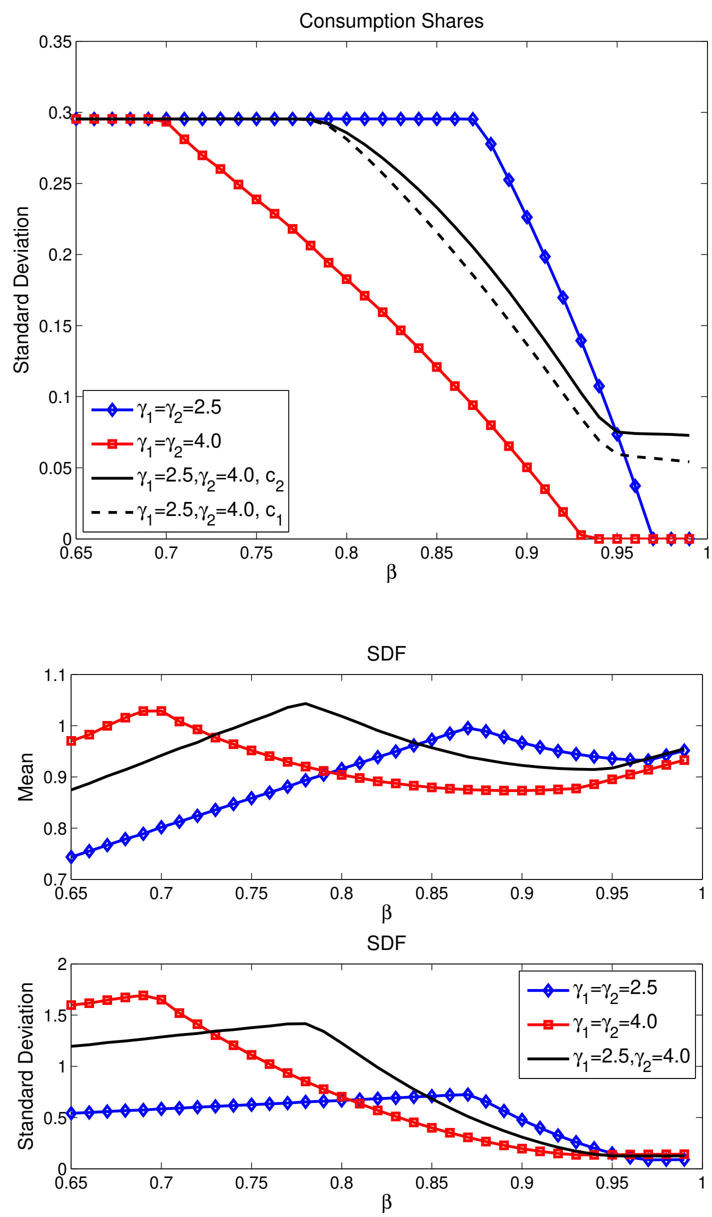


Figure 2.4: Asset Prices: Heterogeneous vs. Homogeneous Risk Aversion

This figure shows how the unconditional moments of the risk-free rate and equity returns change with time discount factor β. Agents have the same β, but may have different risk aversion coefficient γ. The diamond-marked (square-marked) line stands for homogeneous risk aversion $\gamma = 2.5$ ($\gamma = 4.0$), while the solid line stands for heterogeneous risk aversion $\gamma_1 = 2.5$ and $\gamma_2 = 4.0$.

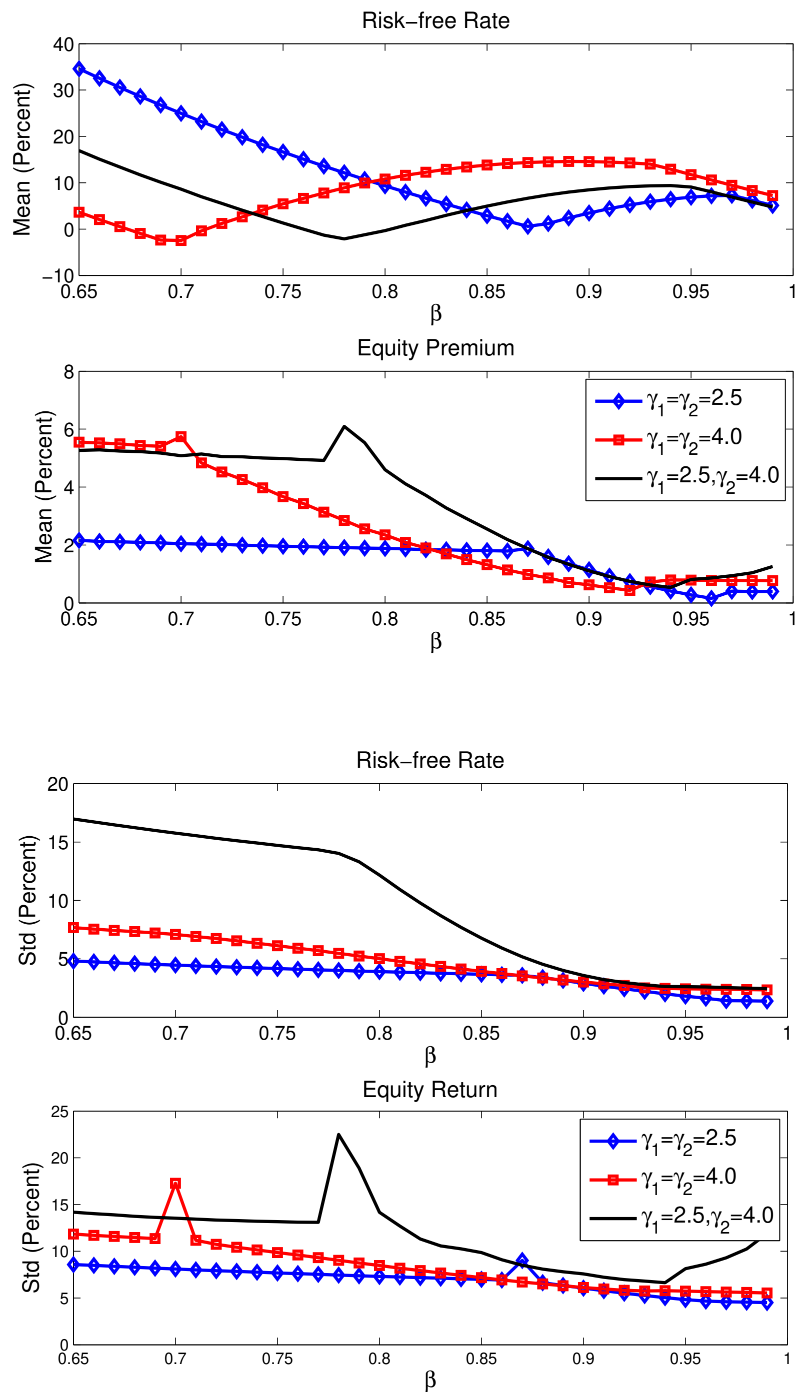
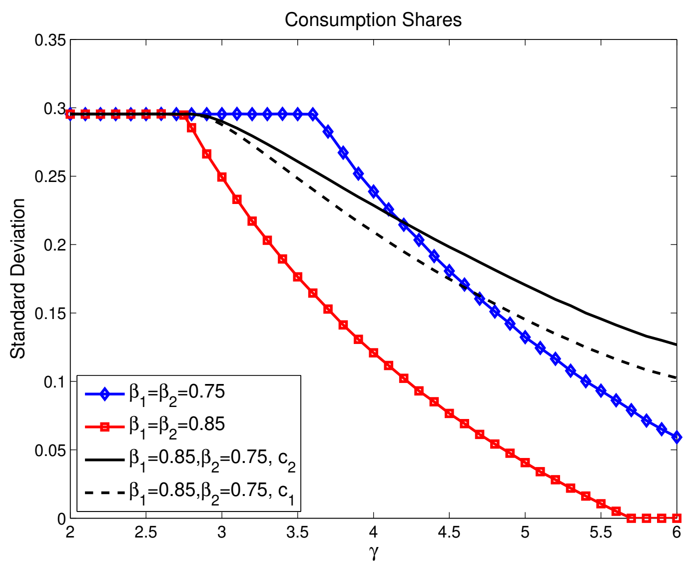


Figure 2.5: Risk Sharing: Heterogeneous vs. Homogeneous Time Discount Factor

This figure shows how agents' consumption shares and the resulting SDF change with risk aversion γ. Agents have the same γ, but may have different time discount factor β. The upper part of the figure presents the standard deviation of consumption shares, which measures the degree of risk sharing. The diamond-marked line indicates that both agents have the same time discount factor $\beta = 0.75$, while the square-marked line the same $\beta = 0.85$. The other two lines present the case of heterogeneous time discount factor $\beta_1 = 0.85$ (the dashed line for agent 1) and $\beta_2 = 0.75$ (the solid line for agent 2). The more patient agent 1 has less volatile consumption than the less patient agent 2 when there is non-zero risk sharing. The lower part of the figure presents the mean and standard deviation of SDF. The diamond-marked (square-marked) line stands for homogeneous time discount factor $\beta = 0.75$ ($\beta = 0.85$), while the solid line stands for heterogeneous time discount factor $\beta_1 = 0.85$ and $\beta_2 = 0.75$.



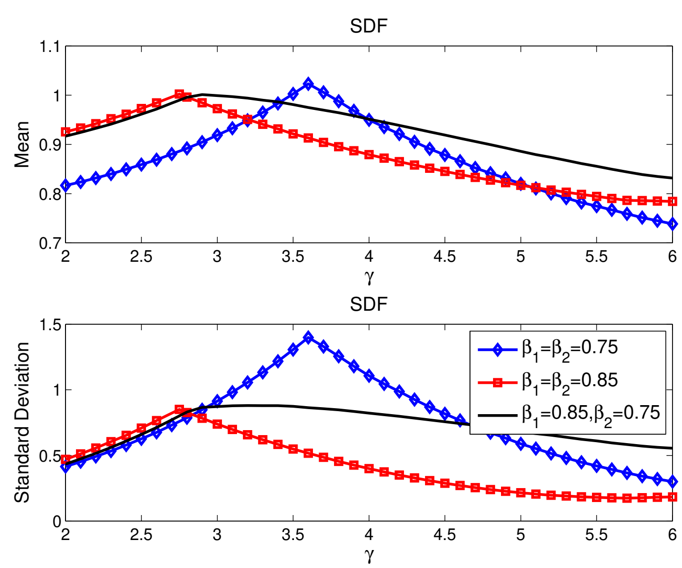


Figure 2.6: Asset Prices: Heterogeneous vs. Homogeneous Time Discount Factor

This figure shows how the unconditional moments of the risk-free rate and equity returns change with risk aversion γ. Agents have the same γ, but may have different time discount factor β. The diamond-marked (square-marked) line stands for homogeneous time discount factor $\beta = 0.75$ ($\beta = 0.85$), while the solid line stands for heterogeneous time discount factor $\beta_1 = 0.75$ and $\beta_2 = 0.85$.

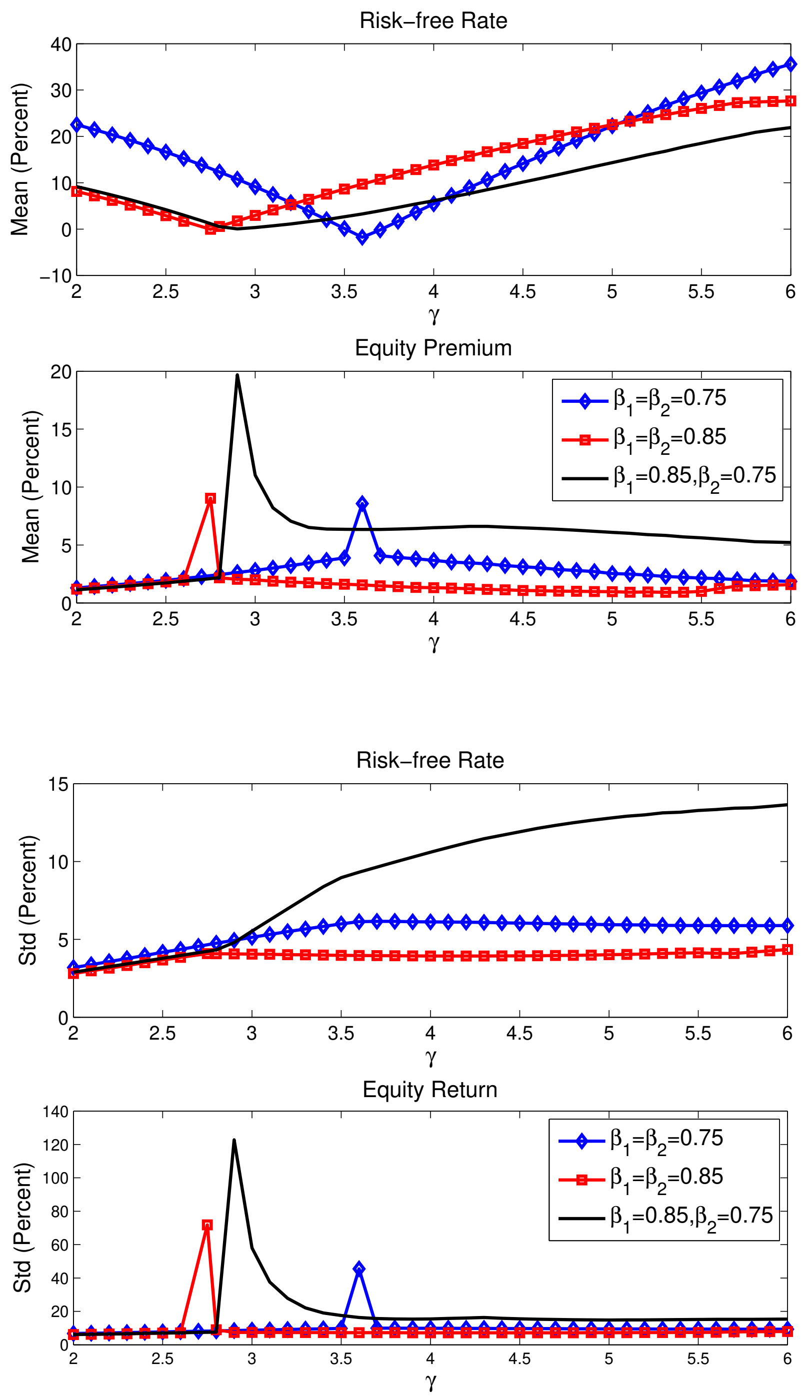


Table 2.1: SDF and Returns Across States: Heterogeneous v.s. Homogeneous Preferences

This table shows the asset prices in the two-state example for each transition from state t to state $t+1$. Panel A fixes $\beta = 0.65$ and compares the case of homogeneous γ with heterogeneous γ. Panel B fixes $\gamma = 3.00$ and compares the case of homogeneous β with heterogeneous β. γ_i is agent i's risk aversion parameter and β_i is agent i's time discount factor. SDF stands for stochastic discount factor, R_s is the (gross) aggregate market return, R_f is the (gross) risk-free rate, and R_e is the equity premium.

Panel A		γ_1	γ_2	β_1	β_2	γ_1	γ_2	β_1	β_2	γ_1	γ_2	β_1	β_2
$\gamma_1 \neq \gamma_2$ vs. $\gamma_1 = \gamma_2$		1.50	2.70	0.65	0.65	1.50	1.50	0.65	0.65	2.70	2.70	0.65	0.65
state t	state $t+1$	SDF	R_s	R_f	R_e	SDF	R_s	R_f	R_e	SDF	R_s	R_f	R_e
1	1	0.65	1.04	1.19	-0.15	0.65	1.14	1.14	0.00	0.65	1.21	1.21	0.00
1	2	1.40	1.41	1.19	0.22	1.55	1.14	1.14	0.00	1.35	1.21	1.21	0.00
2	1	2.62	0.76	0.88	-0.12	1.55	1.14	1.14	0.00	1.35	1.21	1.21	0.00
2	2	0.65	1.03	0.88	0.15	0.65	1.14	1.14	0.00	0.65	1.21	1.21	0.00
Panel B		γ_1	γ_2	β_1	β_2	γ_1	γ_2	β_1	β_2	γ_1	γ_2	β_1	β_2
$\beta_1 \neq \beta_2$ vs. $\beta_1 = \beta_2$		3.00	3.00	0.70	0.50	3.00	3.00	0.50	0.50	3.00	3.00	0.70	0.70
state t	state $t+1$	SDF	R_s	R_f	R_e	SDF	R_s	R_f	R_e	SDF	R_s	R_f	R_e
1	1	0.70	1.08	0.88	0.20	0.50	1.08	1.08	0.00	0.70	1.33	1.33	0.00
1	2	2.45	0.71	0.88	-0.17	2.19	1.08	1.08	0.00	0.90	1.33	1.33	0.00
2	1	0.94	1.73	1.32	0.42	2.19	1.08	1.08	0.00	0.90	1.33	1.33	0.00
2	2	0.70	1.13	1.32	-0.19	0.50	1.08	1.08	0.00	0.70	1.33	1.33	0.00

Table 2.2: Moments for Asset Pricing and Risk Sharing

This table presents the unconditional moments from the model simulations. The moments are averaged across 1,000 simulations, each with 5,000 periods and the first 500 periods burned. $E(R_f)$ ($\sigma(R_f)$) is the mean (volatility) of risk-free rate, $E(R_e)$ or $E(R_s - R_f)$ is the mean of the equity premium, and $\sigma(R_s)$ is the equity return volatility. Sharpe stands for the Sharpe ratio, $E(M)$ ($\sigma(M)$) is the mean (volatility) of SDF, and $\sigma(ln(\hat{c}_i))$ is the volatility of agent i's consumption share. U.S. data sample moments of market excess return, market return volatility, and Sharpe ratio are from Bansal and Yaron (2004). Real risk-free rate sample moments are from Chien and Lustig (2010) using a long sample (1928-2007). AJ denotes the calibration and results from Alvarez and Jermann (2001). Panels A and B show the effects of heterogeneous γ and heterogeneous β, respectively. The preference parameters are chosen around the values from Alvarez and Jermann (2001). For the heterogeneous γ (β) case, risk aversion coefficients are first chosen, and then time discount factor is chosen to match $E(R_f)$ as closely as possible.

	$E(R_f)$	$E(R_e)$	$\sigma(R_f)$	$\sigma(R_s)$	Sharpe	$E(M)$	$\sigma(M)$	$\sigma(ln\hat{c}_1)$	$\sigma(ln\hat{c}_2)$
US	0.89	6.33	4.01	19.42	0.33				
	$\beta = 0.78,\ \gamma = 3.5$								
AJ	0.80	3.41	5.56	9.22	0.37	1.00	1.15	0.276	0.276
	$\beta = 0.88,\ \gamma_1 = \gamma_2 = 2.5$								
Panel A:	1.19	1.59	3.40	6.64	0.24	0.99	0.66	0.278	0.278
$\gamma_1 = \gamma_2$	$\beta = 0.72,\ \gamma_1 = \gamma_2 = 4.0$								
vs.	1.21	4.52	6.73	10.74	0.42	0.99	1.41	0.270	0.270
$\gamma_1 \neq \gamma_2$	$\beta = 0.81,\ \gamma_1 = 2.5, \gamma_2 = 4.0$								
	0.82	4.11	10.93	12.70	0.32	1.00	1.11	0.270	0.277
	$\gamma = 3.75,\ \beta_1 = \beta_2 = 0.75$								
Panel B:	0.87	3.99	6.14	9.94	0.40	0.99	1.29	0.273	0.273
$\beta_1 = \beta_2$	$\gamma = 2.85,\ \beta_1 = \beta_2 = 0.85$								
vs.	1.17	2.09	4.08	7.42	0.28	0.99	0.80	0.275	0.275
$\beta_1 \neq \beta_2$	$\gamma = 3.20,\ \beta_1 = 0.85, \beta_2 = 0.75$								
	1.12	7.05	6.96	27.87	0.25	0.99	0.88	0.273	0.280

Table 2.3: Moments under Independent Risk

This table shows the unconditional moments under independent risk, where aggregate risk is i.i.d. and the distribution of the idiosyncratic risk is independent of aggregate risk (M1=0, M2,8,9,10=1 in the calibration in Table B.1). Moments are averaged across 1,000 simulations, each with 5,000 periods and the first 500 periods burned. $E(R_f)$ ($\sigma(R_f)$) is the mean (volatility) of the risk-free rate, $E(R_s-R_f)$ ($E(R_b-R_f)$) is the mean of the equity premium (the term premium of a perpetual bond), and $\sigma(R_s)$ ($\sigma(R_b)$) is the volatility of the equity return (perpetual bond return). Sharpe stands for the Sharpe ratio, $E(M)$ ($\sigma(M)$) is the mean (volatility) of SDF, and $\sigma(ln(\hat{c}_i))$ is the volatility of agent i's consumption share. $E(R_s)/E(R_f)-1$ is the multiplicative equity premium. Panel A fixes $\beta = 0.5$ and contrasts homogeneous risk aversion of $\gamma = 2.5$ and $\gamma = 3.5$ with heterogeneous risk aversion of $\gamma_1 = 2.5$ and $\gamma_2 = 3.5$. Panel B fixes $\gamma = 3.0$ and contrasts homogeneous time discount factor of $\beta = 0.45$ and $\beta = 0.55$ with heterogeneous time discount factor of $\beta_1 = 0.55$ and $\beta_2 = 0.45$.

	$E(R_f)$	$E(R_b - R_f)$	$E(R_s - R_f)$	$\sigma(R_f)$	$\sigma(R_b)$	$\sigma(R_s)$	Sharpe	$\sigma(ln\hat{c}_1)$	$\sigma(ln\hat{c}_2)$	$\frac{E(R_s)}{E(R_f)} - 1$
Panel A	$\gamma = 2.5$									
$\gamma_1 = \gamma_2$	15.81	0.00	0.36	0.00	0.00	4.07	0.09	0.296	0.296	0.31
vs.	$\gamma = 3.5$									
$\gamma_1 \neq \gamma_2$	24.87	0.00	0.54	0.00	0.00	4.40	0.12	0.196	0.196	0.43
fix $\beta = 0.5$	$\gamma_1 = 2.5,\ \gamma_2 = 3.5$									
	5.96	0.72	1.26	17.03	15.12	16.68	0.08	0.276	0.279	2.44
Panel B	$\beta = 0.45$									
$\beta_1 = \beta_2$	8.19	0.00	0.40	0.00	0.00	3.81	0.10	0.296	0.296	0.37
vs.	$\beta = 0.55$									
$\beta_1 \neq \beta_2$	15.01	0.00	0.43	0.00	0.00	4.05	0.11	0.222	0.222	0.37
fix $\gamma = 3.0$	$\beta_1 = 0.55,\ \beta_2 = 0.45$									
	6.42	4.37	6.77	9.22	25.82	33.02	0.21	0.273	0.277	7.72

Table 2.4: Moments under Positive Assets

This table shows the unconditional moments under positive collateral income. Moments are averaged across 1,000 simulations, each with 5,000 periods and the first 500 periods burned. ω is the share of dividend income as of total income, which is seizable in default. $E(R_f)$ ($\sigma(R_f)$) is the mean (volatility) of the risk-free rate, $E(R_e)$ or $E(R_s - R_f)$ is the mean of equity premium, and $\sigma(R_s)$ is the equity return volatility. Sharpe stands for the Sharpe ratio, and $\sigma(ln(\hat{c}_i))$ is the volatility of agent i's consumption share. Panel A shows the results under homogeneous preferences $\beta = 0.78$ and $\gamma = 3.5$, as in Alvarez and Jermann (2001). Panel B shows the results under heterogeneous risk aversion $\beta = 0.81$, $\gamma_1 = 2.5$ and $\gamma_2 = 4$. Panel C shows the results under heterogeneous time discount factor $\gamma = 3.2$, $\beta_1 = 0.85$, and $\beta_2 = 0.75$.

	ω	$E(R_f)$	$E(R_e)$	$\sigma(R_f)$	$\sigma(R_s)$	Sharpe	$\sigma(ln\hat{c}_1)$	$\sigma(ln\hat{c}_2)$
Panel A:	0.00	0.77	3.45	5.56	9.31	0.37	0.28	0.28
$\beta_1 = \beta_2,\ \gamma_1 = \gamma_2$	0.01	13.93	2.17	4.70	8.18	0.27	0.21	0.21
$\beta = 0.78$	0.05	27.54	1.06	3.46	6.92	0.15	0.10	0.10
$\gamma = 3.5$	0.10	34.28	0.63	2.72	6.28	0.10	0.02	0.02
Panel B:	0.00	0.82	4.11	10.93	12.70	0.32	0.27	0.28
$\gamma_1 \neq \gamma_2$	0.01	11.01	2.41	6.83	9.03	0.27	0.21	0.23
$\beta = 0.81$	0.05	22.37	0.98	3.64	6.75	0.15	0.12	0.14
$\gamma_1 = 2.5,\ \gamma_2 = 4$	0.10	27.85	0.72	3.02	6.47	0.11	0.06	0.08
Panel C:	0.00	1.12	7.05	6.96	27.87	0.25	0.27	0.28
$\beta_1 \neq \beta_2$	0.01	8.43	4.65	8.45	12.32	0.38	0.23	0.25
$\gamma = 3.2$	0.05	17.59	3.35	8.40	11.38	0.29	0.17	0.21
$\beta_1 = 0.85,\ \beta_2 = 0.75$	0.10	21.42	2.59	7.93	10.83	0.24	0.14	0.19

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Appendix A

Chapter 1 Appendices

A.1 The Construction Length of Time

This section shows how construction length of time (LoT) statistics, as in Table 1.3, are constructed. The Census Bureau surveys construction projects, including privately owned nonresidential construction, projects owned by state and local governments, and privately owned multi-family projects, and tracks them from start to completion. It reports the LoT statistics as a supplement to the main estimates of value of construction put in place. LoT statistics are calculated by value and type of construction based on projects completed in a 2-year window. For example, the LoT for all private nonresidential projects during 2014-15 would be the length of time for each project completed in 2014-15[1] weighted by its sampling rate. A sampling rate is assigned to each value-type cell as the inverse of the probability of selecting a project with some adjustments. Sample rates for private nonresidential construction projects are shown in Table 2 in *Construction Methodology* of Construction Spending (`https://www.census.gov/construction/c30/methodology.html`). For example, the sampling rate for manufacturing projects valued at $250,000 to $749,000 is 1/8. As noted by Montgomery (1995), the "equal-weighted" LoT statistics without considering project costs reported by the Census Bureau overstate smaller projects and understate larger projects, distorting the aggregate statistics downward. Thus, I calculate a "value-weighted" version for the sample 2001-2015.

The numbers for 1990-91 in Column 2 of Table 1.3 are taken directly from Census Bureau (1992), except that the row "All (value-weighted)" of 16.7 months is from Montgomery (1995). For the sample 2001-2015, Census Bureau (2016) reports the equal-weighted length of time statistics by

[1] These projects could be started anytime before or during 2014-15.

value and type of construction for each 2-year window, namely 2001-02, 2002-03,...,2013-14, 2014-15. I calculate the value-weighted measures under some assumptions, since the microdata for each project are not observable. Column 2001-15 shows the time-series average. To calculate the row "All (value-weighted)" in Panel A, I assume the average value for each value category equals the midpoint of the range, such as \$2,000 (thousands) for the value category \$1,000 - \$2,999 (thousands). The average value for \$10,000 or more (thousands) is reported by the Census Bureau. I then weight each value category by its average value and number of projects. I assume the distribution of projects to be [1 2 4 8 16 32] for the six value categories (from highest to lowest).[2] For example, the weight for the value category \$1,000 - \$2,999 (thousands) is \$2,000 (thousands) multiplied by 8. The value-weighted length of time statistics for each 2-year window is then calculated as the weighted average across the six value categories. To calculate value-weighted measures for each type of construction in Panel B for 2001-2015, I weight across value categories with their midpoints multiplied by the inverse of the sampling rates mentioned earlier.

A.2 Alternative Construction of Investment Rates

For completeness, this section shows how to calculate investment rates as in Bachmann et al. (2013). I largely follow the description in their paper. Instead of assuming constant depreciation rates and using the perpetual inventory method, I use information on capital stocks and depreciation from BEA FA tables in addition to BEA NIPA tables. The data I use are (i) nominal investment from NIPA Table 1.1.5 Gross Domestic Product at quarterly frequency, $\tilde{I}^Q$, and annual frequency, $\tilde{I}^Y$; (2) investment deflators from NIPA Table 1.1.9 Implicit Price Deflators for Gross Domestic Product at quarterly frequency, P^Q; (3) nominal depreciation from FA Table 1.3. Current-Cost Depreciation of Fixed Assets and Consumer Durable Goods at annual frequency, D^Y; (4) nominal capital stock at year-end prices from FA Table 1.1 Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods at annual frequency, $\tilde{K}^Y$.

First, I construct a quarterly investment series consistent with annual investment. Because original quarterly investment is seasonally adjusted at *annual* rates, the average in each year is not equal to total annual investment. I use $I_t^Q = I_y^Y/4 * \tilde{I}_t{}^Q / \sum_{t \in y} \tilde{I}_t{}^Q$, where y denotes which year. Second, to obtain quarterly depreciation, I assume the real depreciation rate is constant across 4

[2]This is a simple yet reasonable assumption, based on the sampling rates across each value-type cell shown in Table 2 in https://www.census.gov/construction/c30/methodology.html. See the previous paragraph.

quarters in each year and the sum of quarterly depreciation equals annual depreciation,

$$\frac{D_1}{P_1^Q} = \frac{D_2}{P_2^Q} = \frac{D_3}{P_3^Q} = \frac{D_4}{P_4^Q}$$
$$D_1 + D_2 + D_3 + D_4 = D^Y.$$

Third, I adjust annual capital stocks at year-end price to quarter 4 price using $K^Y = \tilde{K}^Y * 2P_{4,y}^Q/(P_{4,y}^Q + P_{1,y+1}^Q)$. I use K^Y as quarter 4 capital stock and use a capital accumulation equation to obtain capital stocks at quarters 1, 2, and 3,

$$K_t^Q = K_{t-1}^Q - D_t^Q + I_t^Q.$$

Finally the real quarterly investment rates are defined as $IK_t^Q = \frac{I_t^Q}{P_t^Q} / \frac{K_{t-1}^Q}{P_{t-1}^Q}$.

A.3 UK Data

Quarterly UK investment data is downloaded from *"gross fixed capital formation by 6 asset types" (namq_pi6_k)* in the Eurostat database.[3] The quantity index for base year 2000 is used, since it has longest time series for the break between equipment and structures. The sample is from 1970Q1 to 2013Q4. Seasonally and calendar adjusted data are used. The 6 asset types are N1111 dwellings, N1112 other buildings and structures, N11131 transport equipment, N11132 other machinery and equipment, N1114 cultivated assets, and N112 intangible fixed assets, along with the aggregate N11 total fixed assets. N1112 is used as the US counterpart of nonresidential structures, and the sum of N11131 and N11132 is taken as nonresidential equipment, which I denote as N1113.

I use the perpetual inventory method to calculate investment rates for equipment and structures, as for US. To calculate the growth rate of total nonresidential equipment N1113, I use the nominal investment (not seasonally adjusted) weighted investment growth rates of N11131 and N11132 (year 2000 index). The depreciation rates used for N1112, N11131, and N11132 are 0.0203, 0.2059, and 0.0757, which are annual and from Oulton and Srinivasan (2003), p.49, Table F ONS2 row.[4] The depreciation rate for N1113 is calculated as the nominal investment (not seasonally adjusted) weighted depreciation rates of N11131 and N11132, resulting in 0.1046.[5]

[3]These data are from the European system of national and regional accounts ESA95. There is an update in September 2014 from ESA95 to ESA 2010, to be consistent with the international System of National Accounts (SNA 2008). I use ESA95, because it has longer time series back to 1970s, while ESA 2010 starts from 1995 for the UK.

[4]These numbers are fairly similar to US numbers.

[5]Ideally, capital stock weighted depreciation should be used. However, Eurostat has only annual capital

Return data are from Kenneth French's and John Campbell's websites and IMF International Financial Statistics.[6] All returns are transformed to log. For nominal stock market return 1970Q1 to 2015Q4, the early sample 1970Q1 to 1974Q4 from Campbell is spliced with the later sample 1975Q1 to 2015Q4 from French.[7] For the nominal 3-month risk-free rate and consumer price index (CPI) from 1964Q1 to 2016Q4, Campbell's data from 1964Q1 to 1996Q4 are directly extended to 2016Q4 using IMF's IFS data, which is Campbell's original source. Each quarter's risk-free rate and CPI are the 3 month Treasury bill yield and Consumer Price Index All items at the quarter-end month, respectively.[8] Realized inflation is the log change in CPI. Real stock return is the nominal stock return minus realized inflation. The ex-post risk-free return is the nominal risk-free rate minus realized inflation.

To obtain ex-ante real risk-free return, I follow the procedure of Beeler and Campbell (2012). I regress the ex-post risk-free return on the risk-free rate (last quarter) and annual realized inflation (divided by 4, last quarter) and use the predicted value as the ex-ante risk-free return.[9] The risk premium is defined as the real stock return minus either the ex-post risk-free return or ex-ante risk-free return.

A.4 Firm Value Derivation

This section shows to how to derive firm value when there is TTB, as shown in equation (1.10). For notational convenience, I denote

$$G(K_{et}, X_{e,t-J_e+1}, K_{st}, X_{s,t-J_s+1}) \equiv G_e(K_{et}, X_{e,t-J_e+1}) + G_s(K_{st}, X_{s,t-J_s+1})$$

$$\Pi(K_{et}, K_{st}) \equiv Y_t - W_t L_t.$$

stock data from 1995, which are derived assuming straight-line depreciation. The depreciation rates from Oulton and Srinivasan (2003) are derived under the assumption of geometric depreciation. By any means, the capital stock weighted depreciation for N1113 is 0.1024 or 0.097, if the 2005 chain-linked volume in national currency or nominal value in national currency (sample period: 1995-2011) is used.

[6] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/F-F_International_Countries.zip, https://dataverse.harvard.edu/dataset.xhtml?persistentId=hdl:1902.1/KSCWRAGNIJ, and http://data.imf.org/?sk=5DABAFF2-C5AD-4D27-A175-1253419C02D1.

[7] Monthly value-weighted market return in local currency without requiring the four price ratios in French's data is first taken log and then summed to quarterly value. The two return series from Campbell and French have a correlation of 0.9978 in the overlapping sample 1975Q1-1997Q1. Early Campbell data 1970Q1-1974Q4 are scaled by the relative ratio of average return from French to average return from Campbell in the overlapping sample.

[8] The IMF's CPI has changed the base year to 2010. I scale the IMF data part 1997Q1-2016Q4 by the relative ratio of Campbell CPI to IMF CPI at 1996Q4.

[9] The regression coefficients on the ex-ante risk-free rate and realized inflation are 0.82 and -0.77, respectively. The only difference from Beeler and Campbell (2012) is that I run the regression at quarterly frequency, while their regression is at monthly frequency.

For further simplification, I will use $G(t)$ and $\Pi(t)$ to denote the above adjustment cost function and revenue function. It is easy to show that both $G(t)$ and $\Pi(t)$ are homogeneous of degree one (HD1). The expression for dividend D_t can then expressed as follows:

$$
\begin{aligned}
D_t =&\Pi(t) - I_{et} - I_{st} - G(t)\\
=&\Pi_{K_e}(t)K_{et} + \Pi_{K_s}(t)K_{st} - \sum_{j=1}^{J_e}\omega_j^e X_{e,t-j+1} - \sum_{j=1}^{J_s}\omega_j^s X_{s,t-j+1}\\
&- G_{K_e}(t)K_{et} - G_{K_s}(t)K_{st} - G_{X_e}(t)X_{e,t-J_e+1} - G_{X_s}(t)X_{s,t-J_s+1}\\
&- q_{et}[K_{e,t+1} - (1-\delta_e)K_{et} - X_{e,t-J_e+1}]\\
&- q_{st}[K_{s,t+1} - (1-\delta_s)K_{st} - X_{s,t-J_s+1}]\\
=&\sum_i^{e,s}\left\{[\Pi_{K_i}(t) - G_{K_i} + q_{it}(1-\delta_i)]K_{it} - q_{it}K_{i,t+1} + [q_{it} - G_{X_i}(t) - \omega_{J_i}^i]X_{i,t-J_i+1} - \sum_{j=1}^{J_i-1}\omega_j^i X_{i,t-j+1}\right\}.
\end{aligned}
$$

The discounted cum-dividend firm value $E_{t-1}(M_{t-1,t}V_t)$ or P_{t-1} can then be derived,

$$
\begin{aligned}
&E_{t-1}(M_{t-1,t}V_t) = E_{t-1}\Big\{M_{t-1,t}[D_t + E_t(M_{t,t+1}D_{t+1}) + ...]\Big\}\\
&= \sum_i^{e,s}\left\{q_{i,t-1}K_{it} - q_{i,t-1}K_{it} + E_{t-1}\Big\{M_{t-1,t}[\Pi_{K_i}(t) - G_{K_i}(t) + q_{it}(1-\delta_i)]K_{it}\Big\}\right\}\\
&+ \sum_i^{e,s} E_{t-1}\Big\{M_{t-1,t}\Big[-q_{it}K_{i,t+1} + E_t\Big(M_{t,t+1}[\Pi_{K_i}(t+1) - G_{K_i}(t+1) + q_{it}(1-\delta_i)]K_{i,t+1}\Big)\Big]\Big\}\\
&- \sum_i^{e,s} E_{t-1}\Big\{M_{t-1,t}E_t\Big(M_{t,t+1}q_{i,t+1}K_{i,t+2}\Big)\Big\} + ...\\
&+ \sum_i^{e,s} E_{t-1}\Big\{M_{t-1,t}\Big([q_{it} - G_{X_i}(t) - \omega_{J_i}^i]X_{i,t-J_i+1} - \sum_{j=1}^{J_i-1}\omega_j^i X_{i,t-j+1}\Big)\Big\}\\
&+ \sum_i^{e,s} E_{t-1}\Big\{M_{t-1,t}E_tM_{t,t+1}\Big([q_{i,t+1} - G_{X_i}(t+1) - \omega_{J_i}^i]X_{i,t+1-J_i+1} - \sum_{j=1}^{J_i-1}\omega_j^i X_{i,t+1-j+1}\Big)\Big\} + ...\\
&= \sum_i^{e,s} q_{i,t-1}K_{it} + \sum_i^{e,s} E_{t-1}\Big\{M_{t-1,t}\Big([q_{it} - G_{X_i}(t) - \omega_{J_i}^i]X_{i,t-J_i+1} - \sum_{j=1}^{J_i-1}\omega_j^i X_{i,t-j+1}\Big)\Big\}\\
&+ \sum_i^{e,s} E_{t-1}\Big\{M_{t-1,t+1}\Big([q_{i,t+1} - G_{X_i}(t+1) - \omega_{J_i}^i]X_{i,t+1-J_i+1} - \sum_{j=1}^{J_i-1}\omega_j^i X_{i,t+1-j+1}\Big)\Big\} + ...,
\end{aligned}
$$

where Euler equations (1.9) are used in the derivation. In standard one-period TTB models, the last two terms will vanish. And $P_{t-1} = \sum_i^{e,s} q_{i,t-1}K_{it}$. Using the above equation, it follows

$$
\begin{aligned}
&E_{t-J_s+1}(M_{t-J_s+1,t}V_t) = E_{t-J_s+1}\left\{M_{t-J_s+1,t-1}E_{t-1}[M_{t-1,t}V_t]\right\}\\
&= E_{t-J_s+1}[M_{t-J_s+1,t-1}(q_{e,t-1}K_{et} + q_{s,t-1}K_{st})]\\
&+ E_{t-J_s+1}\Big[M_{t-J_s+1,t}\Big([q_{st} - G_{X_s}(t) - \omega_{J_s}^s]X_{s,t-J_s+1} - \sum_{j=1}^{J_s-1}\omega_j^s X_{s,t-j+1}\Big)\Big] + ...
\end{aligned}
$$

$$+ E_{t-J_s+1}\Big\{M_{t-J_s+1,t-Je+1}E_{t-J_e+1}\Big[M_{t-J_e+1,t}\Big([q_{et} - G_{X_e}(t) - \omega^e_{J_e}]X_{e,t-J_e+1} - \sum_{j=1}^{J_e-1}\omega^e_j X_{e,t-j+1}\Big)\Big]\Big\} + ...$$

$$= E_{t-J_s+1}[M_{t-J_s+1,t-1}(q_{e,t-1}K_{et} + q_{s,t-1}K_{st})]$$

$$+ E_{t-J_s+1}\left(X_{s,t-J_s+1}\sum_{j=1}^{J_s-1}M_{t-J_s+1,t-J_s+j}\omega^s_j\right)$$

$$+ E_{t-J_s+1}\left(X_{s,t-J_s+2}\sum_{j=1}^{J_s-2}M_{t-J_s+1,t-J_s+j+1}\omega^s_j\right) + ... + E_{t-J_s+1}\left(X_{s,t-1}M_{t-J_s+1,t-1}\omega^s_1\right)$$

$$+ E_{t-J_s+1}\left(X_{e,t-J_e+1}\sum_{j=1}^{J_e-1}M_{t-J_s+1,t-J_e+j}\omega^e_j\right)$$

$$+ E_{t-J_s+1}\left(X_{e,t-J_e+2}\sum_{j=1}^{J_e-2}M_{t-J_s+1,t-J_e+j+1}\omega^e_j\right) + ... + E_{t-J_s+1}\left(X_{e,t-1}M_{t-J_s+1,t-1}\omega^e_1\right),$$

where marginal q equations (1.8) are used in the derivation. Finally, the expected stock price can be derived as follows by shifting the above equation one period forward,

$$E_{t-J_s+2}(M_{t-J_s+2,t}P_t) = E_{t-J_s+2}[M_{t-J_s+2,t}E_t(M_{t,t+1}V_{t+1})]$$

$$= E_{t-J_s+2}(M_{t-J_s+2,t+1}V_{t+1})$$

$$= E_{t-J_s+2}[(M_{t-J_s+2,t}(q_{et}K_{e,t+1} + q_{st}K_{s,t+1})]$$

$$+ E_{t-J_s+2}(X_{s,t-J_s+2}\sum_{j=1}^{J_s-1}M_{t-J_s+2,t-J_s+j+1}\omega^s_j) + ... + E_{t-J_s+2}(X_{st}M_{t-J_s+2,t}\omega^s_1)$$

$$+ E_{t-J_s+2}(X_{e,t-J_e+2}\sum_{j=1}^{J_e-1}M_{t-J_s+2,t-J_e+j+1}\omega^e_j) + ... + E_{t-J_s+2}(X_{et}M_{t-J_s+2,t}\omega^e_1).$$

In my calibration, I assume $J_e = 1$ and $J_s = 5$. The price equation can be written as

$$E_{t-3}(M_{t-3,t}P_t)$$

$$= E_{t-3}[(M_{t-3,t}(q_{et}K_{e,t+1} + q_{st}K_{s,t+1})]$$

$$+ E_{t-3}(X_{s,t-3}\sum_{j=1}^{4}M_{t-3,t-4+j}\omega^s_j) + E_{t-3}(X_{s,t-2}\sum_{j=1}^{3}M_{t-3,t-3+j}\omega^s_j)$$

$$+ E_{t-3}(X_{s,t-1}\sum_{j=1}^{2}M_{t-3,t-2+j}\omega^s_j) + E_{t-3}(X_{st}M_{t-3,t}\omega^s_1).$$

A.5 Additional Results

To better identify the effect of TFP on different types of investment, I estimate separately bivarate VARs with TFP growth (ordered first) and different investment growth rates. Figure A.1 shows the impulse responses (IRFs) of nonresidential equipment investment growth and nonresidential structures investment growth to innovations in TFP growth. When TFP growth increases 1%, equipment investment growth has the largest response on impact, increasing about 1.3%. From quarter 5, it begins to decline and reverts back to steady state in about 20 quarters. The response pattern of structures investment growth is different in the first 4 quarters: It increases about 0.6% percent on impact and persists for 4 quarters. This suggests longer TTB for equipment investment than structures investment.

To complement the results shown in Table 1.4, Table A.1 reports how components of gross private fixed investment predict aggregate risk premium. The residential investment rate shows moderate power for predicting returns. The IPP investment rate has little power to predict returns.

Table A.2 reports how components of government gross investment predict aggregate risk premium. The construction of government investment rates is similar to the construction of private investment rates, as shown in Section 1.2.1. I use real government investment from *NIPA Table 3.9.5* (in nominal value) deflated by *NIPA Table 3.9.4* (price indices). I calculate government capital depreciation rates from the time series average of the ratio of real depreciation (*FA Table 7.3* nominal value in base year 2009 multiplied by *FA Table 7.4* chained quantity indexes) to last-year-end capital stock (*FA Table 7.1* nominal value in base year 2009 multiplied by *FA Table 7.2* chained quantity indexes). With real investment series and depreciation rates, I use the perpetual inventory method in equation (1.1) to calculate government investment rates. Government investment, especially equipment investment, shows positive prediction for stock returns.

Figure A.1: Impulse Response Functions (IRFs) of Investment Growth to TFP Growth Innovation

This figure shows the impulse responses of nonresidential equipment investment growth (left panel) and nonresidential structures investment growth (right panel) to innovations in TFP growth, generated by separately estimating bivariate VARs with TFP growth (ordered first) and different investment growth rates. Shaded areas are one standard error confidence bands from Kilian's (1998) bootstrap-after-bootstrap. The sample period is 1947Q1-2015Q4.

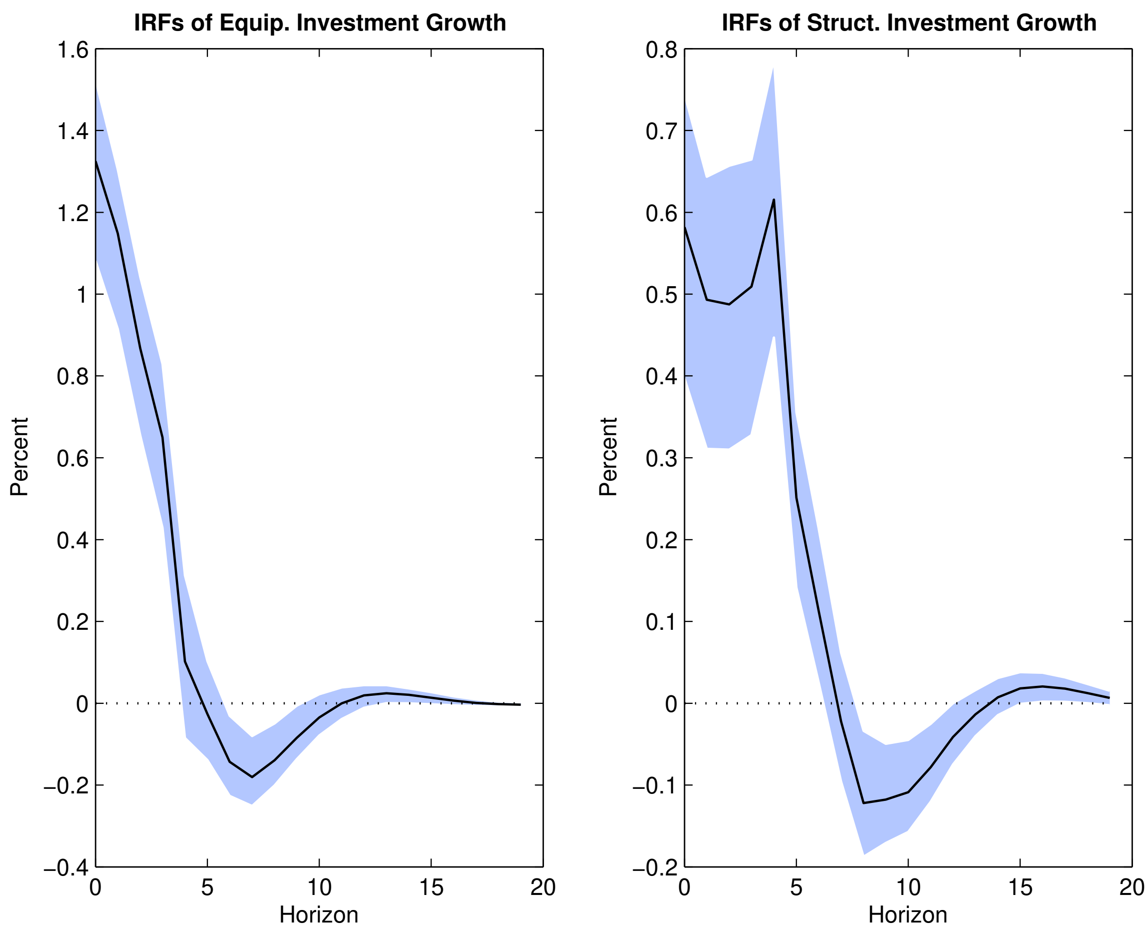

Table A.1: Return Predictability from Private Investment

This table reports in-sample and out-of-sample R^2 (in percent) for OLS predictions of US aggregate risk premium (from Kenneth French's website) from 1947Q1 to 2015Q4 across various horizons (H) ranging from 1 quarter to 20 quarters, $\sum_{h=1}^{H} R_{t+h} = a + b\ \mathrm{IK}_t + \varepsilon_{t+H}$. Predictor variables are US investment rates of nonresidential total (including intellectual property and products (IPP)), nonresidential IPP, residential, and gross private fixed including both nonresidential and residential. The out-of-sample procedure uses the first half of the sample as the training period, then recursively tests and retrains in subsequent periods. b denotes the prediction slope coefficient. *p(NW)* denotes in-sample p-values constructed as in Newey and West (1987). Out-of-sample R^2 is calculated against historical averages of the predicted variable. *ENC-NEW* denotes the *New Encompassing* out-of-sample test statistic from Clark and McCracken (2001), following the construction methodology described in Kelly and Pruitt (2013). Significance for ENC-NEW statistics: $***: p < 0.01$, $**: p < 0.05$, $*: p < 0.1$.

Investment Rates	H	In Sample			Out of Sample	
		$R^2\%$	b	$p(NW)$	$R^2\%$	ENC-NEW
Nonresidential including IPP	1	3.76	-6.40	0.002	0.32	2.833**
	4	11.04	-22.48	0.001	5.38	3.961***
	8	18.60	-39.84	0.000	14.97	4.815***
	12	29.76	-58.89	0.000	27.80	7.341***
	16	39.29	-75.32	0.000	36.12	9.877***
	20	41.21	-88.82	0.000	29.70	9.621***
IPP	1	0.13	-0.59	0.548	-0.39	-0.235
	4	0.74	-2.93	0.364	-0.41	-0.049
	8	1.34	-5.44	0.423	-1.49	-0.157
	12	3.27	-10.07	0.290	0.38	0.118
	16	5.76	-15.10	0.178	4.01	0.517
	20	6.43	-18.54	0.170	4.84	0.618
Residential	1	0.49	-2.35	0.250	-1.41	-0.167
	4	2.54	-11.08	0.084	-5.46	0.353
	8	6.14	-24.10	0.022	-0.92	1.822**
	12	11.40	-39.31	0.001	7.79	3.155**
	16	11.54	-45.67	0.001	19.03	3.245***
	20	11.78	-55.76	0.004	22.01	3.154**
Gross Private	1	2.79	-6.84	0.009	-2.31	3.028**
	4	9.50	-26.05	0.001	-1.79	4.595***
	8	18.04	-49.58	0.000	8.46	6.415***
	12	31.07	-77.12	0.000	23.89	10.338***
	16	38.36	-97.31	0.000	40.03	15.220***
	20	41.29	-120.26	0.000	46.31	24.504***

Table A.2: Return Predictability from Government Investment

This table reports in-sample R^2 (in percent) for OLS predictions of US aggregate risk premium (from Kenneth French's website) across various horizons (H) ranging from 1 quarter to 20 quarters, $\sum_{h=1}^{H} R_{t+h} = a + b\ \mathrm{IK}_t + \varepsilon_{t+H}$. Predictor variables are US investment rates from government, including gross investment and its components, equipment, structures, and IPP. The whole sample is 1947Q1-2015Q4. The sample in Jones and Tuzel (2013b) is 1958Q1-2009Q4. b denotes the prediction slope coefficient. *p(NW)* denotes in-sample p-values constructed as in Newey and West (1987).

Investment Rates	H	Sample: 1947Q1-2015Q4			Sample: 1958Q1-2009Q4		
		$R^2\%$	b	$p(NW)$	$R^2\%$	b	$p(NW)$
Gross	1	0.70	0.91	0.079	0.07	1.42	0.706
	4	2.77	3.73	0.070	0.31	5.82	0.613
	8	6.41	7.79	0.028	0.35	7.99	0.669
	12	10.42	11.62	0.005	0.14	5.65	0.805
	16	12.70	14.27	0.004	0.05	3.52	0.884
	20	13.14	16.52	0.003	0.00	0.50	0.985
Equipment	1	0.65	0.36	0.078	0.33	0.67	0.472
	4	3.09	1.62	0.080	1.62	2.96	0.311
	8	9.20	3.80	0.038	4.35	6.33	0.174
	12	18.40	6.25	0.000	6.34	8.65	0.103
	16	23.60	7.83	0.000	10.76	11.98	0.016
	20	24.14	8.96	0.000	14.31	15.45	0.002
Structures	1	0.56	1.03	0.127	0.09	-2.86	0.634
	4	1.77	3.75	0.141	0.42	-12.22	0.536
	8	3.22	6.95	0.129	2.10	-34.95	0.173
	12	4.69	9.85	0.117	4.43	-57.12	0.032
	16	5.79	12.22	0.139	9.04	-86.29	0.001
	20	6.31	14.56	0.153	13.11	-115.95	0.000
IPP	1	0.09	0.26	0.581	0.01	0.11	0.901
	4	0.25	0.90	0.563	0.00	0.07	0.979
	8	0.51	1.76	0.492	0.02	-0.52	0.919
	12	0.57	2.17	0.549	0.20	-1.89	0.788
	16	0.58	2.44	0.590	0.57	-3.35	0.690
	20	0.41	2.33	0.651	1.43	-5.91	0.503

Appendix B

Chapter 2 Appendices

B.1 Computation

Let state variables be $x = (\lambda, z)$. The policy and value functions are $\hat{c}_i(x), \lambda'(x), \nu_i(x), W_i(x)$, where the value function

$$W_i(x) = u(\hat{c}_i(x)) + \beta_i(z) \sum_{z'} \pi_i(z'|z) W_i(x'). \tag{B.1}$$

The computation algorithm follows these steps:

1. Set up a grid X over the state space.

2. Set the initial guess to be the solution to the planner's problem without enforcement constraints. $\nu_i^0(x) = 0, z'^0(x) = z$, $\hat{c}_i^0(x)$, and $W_i^0(x)$ satisfy (2.10), (2.13) and (B.1).

3. Consider three possible binding patterns of enforcement constraints:

- Neither constraint binds
- Agent 1's constraint binds
- Agent 2's constraint binds

3.1 For each $x \in X$, compute allocations that assume neither constraint binds.

3.2 Then check

$$u(\hat{c}_i^0(x)) + \beta_i(z) \sum_{z'} \pi(z'|z) W_i^0(x') \geq U^i(\hat{e}_i(z)) \quad \text{for } i = 1, 2. \tag{B.2}$$

3.2.1 If (B.2) is satisfied for $i = 1, 2$, then set new policies

$$\nu_i^1(x) = \nu_i^0(x), \lambda'^1(x) = \lambda'^0(x), \hat{c}_i^1(x) = \hat{c}_i^0(x), W_i^1(x) = W_i^0(x).$$

3.2.2 If (B.2) is satisfied for $i = 2$ but not $i = 1$, then set $\nu_2^1(x) = 0$,

solve $\nu_1^1(x), \lambda'^1(x), \hat{c}_1^1(x)$, and $\hat{c}_2^1(x)$ from (2.10), (2.11), (2.13) and

$$u^1(\hat{c}_1^1(x)) + \beta(z) \sum_{z'} \pi(z'|z) W_1^1(x') = U^1(\hat{e}_1(z)). \tag{B.3}$$

Set $W_1^1(x)$ as the LHS (left-hand-side) of (B.3) and $W_2^1(x)$ as the LHS of (B.2).

3.2.3 If (B.2) is satisfied for $i = 1$ but not $i = 2$, then set $\nu_1^1(x) = 0$,

solve $\nu_2^1(x), \lambda'^1(x), \hat{c}_1^1(x)$, and $\hat{c}_2^1(x)$ from (2.10), (2.11), (2.13) and

$$u^2(\hat{c}_2^1(x)) + \beta(z) \sum_{z'} \pi(z'|z) W_2^1(z') = U^2(\hat{e}_2(z)). \tag{B.4}$$

Set $W_1^1(x)$ as the LHS of (B.2) and $W_2^1(x)$ as the LHS of (B.4).

4.1 If the difference between $(\nu_i^0(x), \lambda'^0(x), \hat{c}_i^0(x), W_i^0(x))$ and $(\nu_i^1(x), \lambda'^1(x), \hat{c}_i^1(x), W_i^1(x))$ is small enough for each $x \in X$, then stop.

4.2 If not, then set the initial guess equal to the new set of policy, multiplier, and value functions. Keep iterating until the value functions and policy functions converge.

B.2 Calibration

The endowment process is

State	g	$\hat{e}_2$	$\hat{e}_1$
1	g_L	$\frac{1}{2}-\theta$	$\frac{1}{2}+\theta$
2	g_H	$\frac{1}{2}-\eta$	$\frac{1}{2}+\eta$
3	g_L	$\frac{1}{2}+\theta$	$\frac{1}{2}-\theta$
4	g_H	$\frac{1}{2}+\eta$	$\frac{1}{2}-\eta$

The transition matrix is

$$\begin{bmatrix} p_{LL}\pi_{LL} & p_{LH}(1-\pi_{LL}) & (1-p_{LL})\pi_{LL} & (1-p_{LH})(1-\pi_{LL}) \\ p_{HL}(1-\pi_{HH}) & p_{HH}\pi_{HH} & (1-p_{HL})(1-\pi_{HH}) & (1-p_{HH})\pi_{HH} \\ (1-p_{LL})\pi_{LL} & (1-p_{LH})(1-\pi_{LL}) & p_{LL}\pi_{LL} & p_{LH}(1-\pi_{LL}) \\ (1-p_{HL})(1-\pi_{HH}) & (1-p_{HH})\pi_{HH} & p_{HL}(1-\pi_{HH}) & p_{HH}\pi_{HH} \end{bmatrix}$$

Aggregate growth rate follows a Markov process, and the idiosyncratic income shares follow a Markov process conditional on the aggregate transition. π_{LL} (π_{HH}) denotes the aggregate transition probability from recession (boom) to recession (boom). p_{ij} denotes the idiosyncratic transition probability of agents having the same relative status (higher or lower than the other agent) of income share conditional on the aggregate state transition from i to j.

There are 10 parameters to be calibrated in addition to the preference parameters:

$$g_L,\ g_H,\ \theta,\ \eta,\ \pi_{LL},\ \pi_{HH},\ p_{LL},\ p_{LH},\ p_{HL},\ p_{HH}.$$

The calibrated parameters, following Alvarez and Jermann (2001), are in Table B.1.

The resulting endowment process is

State	g	$\hat{e}_2$	$\hat{e}_1$
1	0.9602	0.3562	0.6438
2	1.0402	0.3562	0.6438
3	0.9602	0.6438	0.3562
4	1.0402	0.6438	0.3562

And the resulting transition matrix is

$$\begin{bmatrix} 0.1414 & 0.8200 & 0.0309 & 0.0077 \\ 0.2637 & 0.6820 & 0.0486 & 0.0057 \\ 0.0309 & 0.0077 & 0.1414 & 0.8200 \\ 0.0486 & 0.0057 & 0.2637 & 0.6820 \end{bmatrix}$$

Table B.1: Parameter Values

Parameter	Symbol	Value	Moments
Aggregate growth rate in recessions (L)	g_L	0.9602	Mehra-Prescott (1985): M1: $\rho(g) = -0.14$
Aggregate growth rate in booms (H)	g_H	1.0402	M2: $E(g) = 1.83\%$
Transition probability from L to L	π_{LL}	0.1723	M3: $Std(g) = 3.57\%$
Transition probability from H to H	π_{HH}	0.6877	NBER 1889-1991: M4: Pr(boom)/Pr(recession) = 2.65
High income share in recessions	$1/2 + \theta$	0.6438	Alvarez-Jermann (2001):
High income share in booms	$1/2 + \eta$	0.6438	M5: $Std(\ln \hat{e}_i(z)) = 0.296$
Idiosyncratic transition probability of agent 2 low-low (high-high) income shares conditional on LL	p_{LL}	0.8206	M6: $\rho(\ln \hat{e}_i(z)) = 0.9$ M7: $\sum_{i=1,2} \left[\hat{e}_i(L) - \frac{1}{2}\right]^2 = \sum_{i=1,2} \left[\hat{e}_i(H) - \frac{1}{2}\right]^2$
Idiosyncratic transition probability of agent 2 low-low (high-high) income shares conditional on HH	p_{HH}	0.9907	M8: $\frac{\sigma_{L'H}}{\sigma_{H'H}} = \frac{std(\ln \hat{e}_i(z_{t+1}) \vert g_{t+1} = g_L, g_t = g_H)}{std(\ln \hat{e}_i(z_{t+1}) \vert g_{t+1} = g_H, g_t = g_H)} = 4$
Idiosyncratic transition probability of agent 2 low-low (high-high) income shares conditional on LH	p_{LH}	0.8444	M9: $\frac{\sigma_{L'L}}{\sigma_{H'L}} = \frac{std(\ln \hat{e}_i(z_{t+1}) \vert g_{t+1} = g_L, g_t = g_L)}{std(\ln \hat{e}_i(z_{t+1}) \vert g_{t+1} = g_H, g_t = g_L)} = 4$
Idiosyncratic transition probability of agent 2 low-low (high-high) income shares conditional on HL	p_{HL}	0.9917	M10: $\frac{\sigma_L}{\sigma_H} = \frac{std(\ln \hat{e}_i(z_{t+1}) \vert g_t = g_L)}{std(\ln \hat{e}_i(z_{t+1}) \vert g_t = g_H)} = 0.85$